Skill Builder: Topics 2.1-2.3 – The Definition of the Derivative

Compute the derivative function, $f'(x)$, using the definition of derivative for each of the following.
<u>Use proper notation.</u>

1. $f(x) = 3x^2 + 1$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\left[3(x+h)^2+1\right] - \left(3x^2+1\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left[3\left(x^2+2xh+h^2\right)\right] - \left(3x^2\right)}{h} = \lim_{h\to 0}\frac{\left(3x^2+6xh+3h^2\right) - \left(3x^2\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left(6xh+3h^2\right)}{h} = \lim_{h\to 0}\frac{\left(6x+3h\right)}{1} = 6x$$

2. $f(x) = 2$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{(2)-(2)}{h}$$

$$= \lim_{h\to 0}\frac{(2)-(2)}{h} = 0$$

3. $f(x) = \dfrac{3}{x+1}$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\left[\dfrac{3}{(x+h)+1}\right] - \left(\dfrac{3}{x+1}\right)}{h} = \lim_{h\to 0}\frac{3(x+1) - 3(x+h+1)}{h(x+h+1)(x+1)} = \lim_{h\to 0}\frac{3x+3-3x-3h-3}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0}\frac{-3h}{h(x+h+1)(x+1)} = \lim_{h\to 0}\frac{-3}{(x+h+1)(x+1)} = \frac{-3}{(x+1)^2}$$

4. $f(x) = \sqrt{3x+1}$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h}$$

$$= \lim_{h\to 0}\frac{\left[\sqrt{3(x+h)+1} - \sqrt{3x+1}\right]\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]}{h\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]} = \lim_{h\to 0}\frac{\left[\left(3(x+h)+1\right) - \left(3x+1\right)\right]}{h\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]}$$

$$= \lim_{h\to 0}\frac{\left[\left(3x+3h+1\right) - \left(3x+1\right)\right]}{h\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]} = \lim_{h\to 0}\frac{3h}{h\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]}$$

$$= \lim_{h\to 0}\frac{3}{\left[\sqrt{3(x+h)+1} + \sqrt{3x+1}\right]} = \frac{3}{2\sqrt{3x+1}}$$

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Compute $f'(c)$ for each of the following using the alternate form of the definition of the derivative.

5. $f(x)=3x+1,\ c=1$ $f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$ $=\lim_{h\to 0}\frac{3(1+h)+1-(4)}{h}=\lim_{h\to 0}\frac{3+3h+1-4}{h}$ $=\lim_{h\to 0}\frac{3h}{h}=\lim_{h\to 0}\frac{3}{1}=3$	**6.** $f(x)=x+\frac{4}{x},\ c=4$ $f'(4)=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$ $=\lim_{h\to 0}\frac{\left[(4+h)+\frac{4}{(4+h)}\right]-(5)}{h}$ $=\lim_{h\to 0}\frac{\left[(4+h)^2+4\right]-(5)(4+h)}{h(4+h)}$ $=\lim_{h\to 0}\frac{\left[(4+h)^2-5h-16\right]}{h(4+h)}$ $=\lim_{h\to 0}\frac{\left[16+8h+h^2-5h-16\right]}{h(4+h)}$ $=\lim_{h\to 0}\frac{\left[3h+h^2\right]}{h(4+h)}=\lim_{h\to 0}\frac{\left[3+h\right]}{(4+h)}=\frac{3}{4}$

For Problem 7 & 8, the given limits represent an $f'(c)$ for a function $f(x)$ and a number c. Find f and c.

7. $\lim_{\Delta x\to 0}\frac{\left[5-3(1+\Delta x)\right]-2}{\Delta x}$ $f(x)=5-3x \qquad c=1$	**8.** $\lim_{\Delta x\to 0}\frac{(-2+\Delta x)^3+8}{\Delta x}$ $f(x)=x^3 \qquad c=-2$

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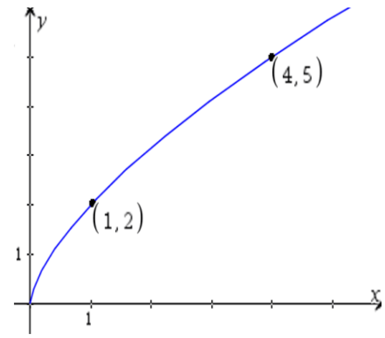
For Problems 9 and 10, consider the sketch of the function, $f(x)$, to the right.



9. a.) Find $f(1)$ and $f(4)$. $f(1) = 2 \qquad f(4) = 5$	**b.)** Calculate $f(4) - f(1)$. $f(4) - f(1) = 3$

c.) What does the equation $y = \dfrac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$ represent?

This is the equation of the secant line joining the two points (1, 2) and (4, 5).

10. Insert the proper inequality symbol (< or >) between the given quantities. If you need help, draw the represented line in the picture above

a.) $\dfrac{f(4) - f(1)}{4 - 1}$ [>] $\dfrac{f(4) - f(3)}{4 - 3}$	**b.)** $\dfrac{f(4) - f(1)}{4 - 1}$ [<] $f'(1)$

11. The table shows the margin of error in degrees for tennis serves hit at 100 mph with various amounts of topspin (in units of revolutions per second).

Topspin (rps) (x)	20	40	60	80	100
Margin of error $f(x)$	1.8	2.4	3.1	3.9	4.6

a.) Estimate the derivative of f at $x = 70$ rps. Label correctly.

$$f'(70) \approx \frac{f(80) - f(60)}{80 - 60} = \frac{(3.9) - (3.1)}{20} = \frac{0.8}{20} = 0.04$$

b.) Estimate the derivative of f at $x = 40$ rps. Label correctly.

$$f'(40) \approx \frac{f(60) - f(20)}{60 - 20} = \frac{(3.1) - (1.8)}{40} = \frac{1.3}{40} = 0.0325$$