Skill Builder: Topic 2.10 – Derivatives of tan*x*, cot*x*, sec*x*, csc*x*   Acrostic

When completed properly, the table below will reveal a portion of a quote made famous by one of the founders of calculus. To unveil the letters, answer each multiple choice question correctly and place the appropriate letter in the square that corresponds to the question number. Some problem numbers may appear more than once.

12	8	8	10	8	9		13	8	12		4	10	11				
E	R	R	O	R	S		A	R	E		N	O	T				
5	4		11	7	12		13	8	11		2	3	11				
I	N		T	H	E		A	R	T		B	U	T				
5	4		11	7	12		13	8	11	5	6	5	1	12	8	9	
I	N		T	H	E		A	R	T	I	F	I	C	E	R	S	

1. Find $f'(x)$ for $f(x) = \tan x + \cos x$.

(A) $\sec x - \sin x$ **(B)** $\sec^2 x + \sin x$ **(C)** $\sec^2 x - \sin x$ **(D)** $\dfrac{\sin x}{\cos x} - \sin x$

$f'(x) = \sec^2 x + (-\sin x) = \sec^2 x - \sin x$

2. Find $f'(t)$ for $f(t) = t^2 \tan t$.

(D) $2t\tan t - t^2 \sec t$ **(B)** $t(2\tan t + t\sec^2 t)$ **(G)** $2t\sec^2 t$ **(T)** $t(2\tan t + t\sec t)$

$f'(t) = 2t\tan t + t^2\sec^2 t = t\left(2\tan t + t\sec^2 t\right)$

3. Find $\dfrac{dy}{dx}$ for $y = e^x \sec x$.

(U) $e^x \sec x(1+\tan x)$ **(O)** $e^x \sec x \tan x$ **(A)** $e^x \tan x + e^x \sec x \tan x$ **(E)** $e^x(\sec x + \tan^2 x)$

$\dfrac{dy}{dx} = e^x \sec x + e^x \sec x \tan x = e^x \sec x\left(1 + \tan x\right)$

4. Find $\dfrac{dy}{du}$ for $y = \pi u \cdot \tan u$.

(T) $\pi \sec^2 u$ **(S)** $\pi \tan u - \pi u \cdot \sec^2 u$ **(N)** $\pi\left(\tan u + u \cdot \sec^2 u\right)$ **(F)** $\pi\left(\tan u + u \cdot \sec u\right)$

$\dfrac{dy}{du} = \pi \tan u + \pi u \cdot \sec^2 u = \pi\left(\tan u + u \cdot \sec^2 u\right)$

5. Find $f'(\theta)$ for $f(\theta) = \csc\theta \cot\theta$.

(A) $-\csc\theta\left(\cot^2\theta - \csc^2\theta\right)$ **(E)** $-\csc\theta$ **(I)** $-\csc\theta\left(\cot^2\theta + \csc^2\theta\right)$ **(O)** $-\csc\theta\cot\theta - \csc^2\theta$

$f'(\theta) = \left(-\csc\theta\cot\theta\right)\cot\theta + \csc\theta\left(-\csc^2\theta\right) = -\csc\theta\left(\cot^2\theta + \csc^2\theta\right)$

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6. Find the equation of the tangent line to the graph of $f(x)=\tan x$ at the point $\left(\frac{\pi}{4},1\right)$.

(C) $y=1+\sqrt{2}\left(x-\frac{\pi}{4}\right)$ **(D)** $y=1+\frac{1}{2}\left(x-\frac{\pi}{4}\right)$ **(N)** $y=-1+2\left(x-\frac{\pi}{4}\right)$ **(F)** $y=1+2\left(x-\frac{\pi}{4}\right)$

$$f'\left(\frac{p}{4}\right)=\sec^2\left(\frac{p}{4}\right)=\left(\sqrt{2}\right)^2=2\Rightarrow y=1+2\left(x-\frac{p}{4}\right)$$

7. Find $\frac{dy}{dx}$ for $y=4e^x\cot x$.

(R) $4xe^{x-1}\cot x-4e^x\csc^2 x$ **(T)** $4e^x\left(\csc^2 x+\cot x\right)$

(H) $4e^x\left(\cot x-\csc^2 x\right)$ **(N)** $4e^x\left(\csc^2 x-\cot x\right)$

$$\frac{dy}{dx}=4e^x\cot x+4e^x\left(-\csc^2 x\right)=4e^x\left(\cot x-\csc^2 x\right)$$

8. Find $\frac{dy}{dx}$ for $y=2x-x^2\tan x$

(P) $2-2x\sec^2 x$ **(R)** $2-x^2\sec^2 x-2x\tan x$

(H) $2-x^2\sec^2 x+2x\tan x$ **(T)** $2-x^2\sec^2 x-2x\sec x\tan x$

$$\frac{dy}{dx}=2-\left(2x\tan x+x^2\sec^2 x\right)=2-2x\tan x-x^2\sec^2 x$$

9. Find $\frac{dy}{dx}$ for $y=4xe^x-\cot x$

(E) $4xe^x+4e^x-\csc^2 x$ **(R)** $4e^x-\csc^2 x$

(S) $4xe^x+4e^x+\csc^2 x$ **(T)** $4xe^x+4e^x+\csc x\cot x$

$$\frac{dy}{dx}=4e^x+4xe^x-\left(-\csc^2 x\right)=4e^x+4xe^x+\csc^2 x$$

10. Which of the following is equivalent to $\frac{dy}{dx}$ for $y=x-\tan x$

(A) $\tan^2 x$ **(E)** $1+\sec^2 x$ **(I)** $-\sec^2 x$ **(O)** $-\tan^2 x$

$$\frac{dy}{dx}=1-\sec^2 x=-\tan^2 x$$

11. Find $\frac{dy}{dx}$ for $y=\frac{\cot x}{1+\cot x}$

(T) $\frac{-\csc^2 x}{(1+\cot x)^2}$ **(S)** $\frac{2\csc^2 x}{(1+\cot x)^2}$

(W) $\frac{-\csc^2 x}{1+\cot^2 x}$ **(Y)** $\frac{\csc^2 x}{(1+\cot x)^2}$

$$\frac{dy}{dx}=\frac{(1+\cot x)\left(-\csc^2 x\right)-\left(-\csc^2 x\right)\cot x}{(1+\cot x)^2}=\frac{\left(-\csc^2 x-\csc^2 x\cot x\right)+\left(\csc^2 x\cot x\right)}{(1+\cot x)^2}=\frac{-\csc^2 x}{(1+\cot x)^2}$$

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12. Which of the following is equivalent to $\lim_{h\to 0}\frac{\csc(x+h)-\csc x}{h}$.

(A) $-\csc^2 x$ **(E)** $-\csc x\cot x$ **(I)** $-\sec x\tan x$ **(O)** 0

$$\lim_{h\to 0}\frac{\csc(x+h)-\csc x}{h}=\frac{d}{dx}\csc x=-\csc x\cot x$$

13. Find $\frac{dy}{dx}$ for $y=\frac{\sec x}{1+\tan x}$

(E) $\frac{\tan^2 x+\tan^3 x-\sec^3 x}{(1+\tan x)^2}$ **(T)** $\frac{\tan x}{\sec x}$

(W) $\frac{\sec x+\sec x\tan^2 x-\sec^3 x}{(1+\tan x)^2}$ **(A)** $\frac{\sec x(\tan x+\tan^2 x-\sec^2 x)}{(1+\tan x)^2}$

$$\frac{dy}{dx}=\frac{(1+\tan x)(\sec x\tan x)-(\sec^2 x)\sec x}{(1+\tan x)^2}=\frac{\sec x(\tan x+\tan^2 x-\sec^2 x)}{(1+\tan x)^2}$$

14. Who originally stated the quote and what does it mean?

The quote is by Isaac Newton. The error is not in the theory but in the application of the theory.