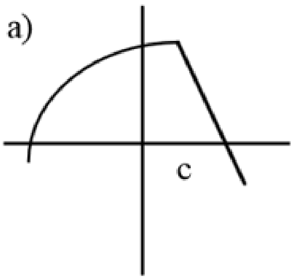
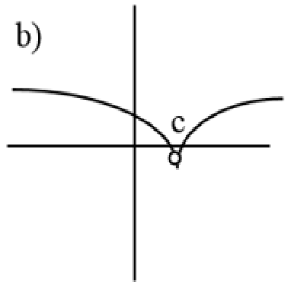
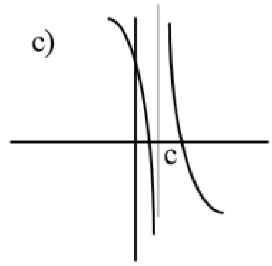
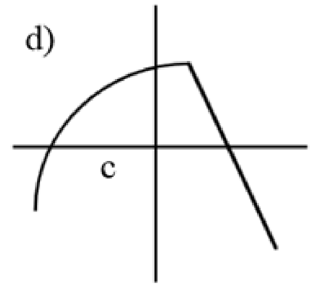
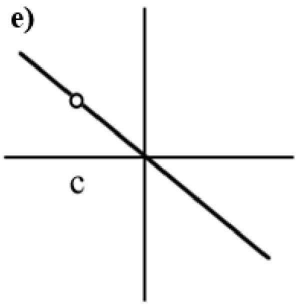
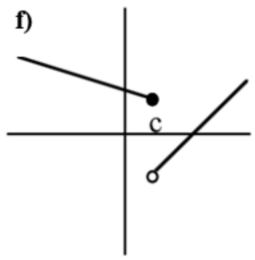
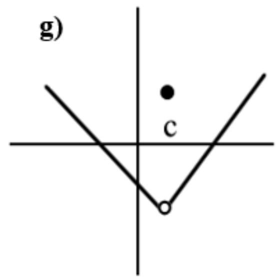
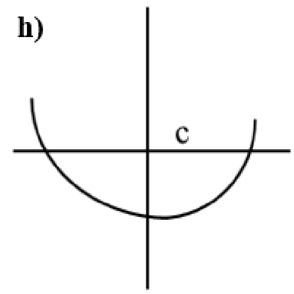
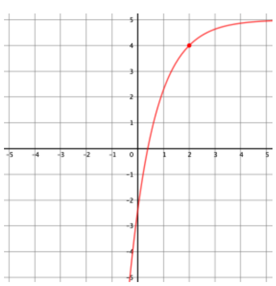
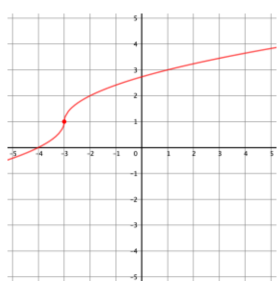
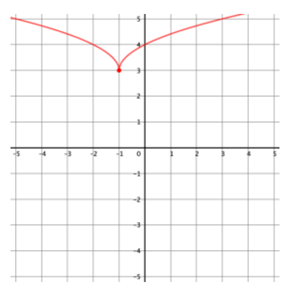
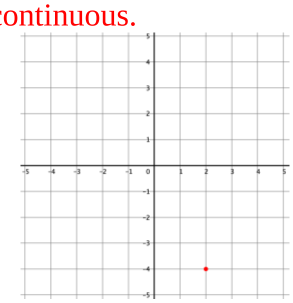
Skill Builder: Topics 2.4 – Differentiability

1.) For the following, state whether the function is continuous, differentiable, both or neither at *x* = *c*.

a)  Continuous and not differentiable.	b)  Not continuous and not differentiable.	c)  Not continuous and not differentiable.	d)  Continuous and differentiable.
e)  Not continuous and not differentiable.	**f)**  Not continuous and not differentiable.	**g)**  Not continuous and not differentiable.	**h)**  Continuous and differentiable.

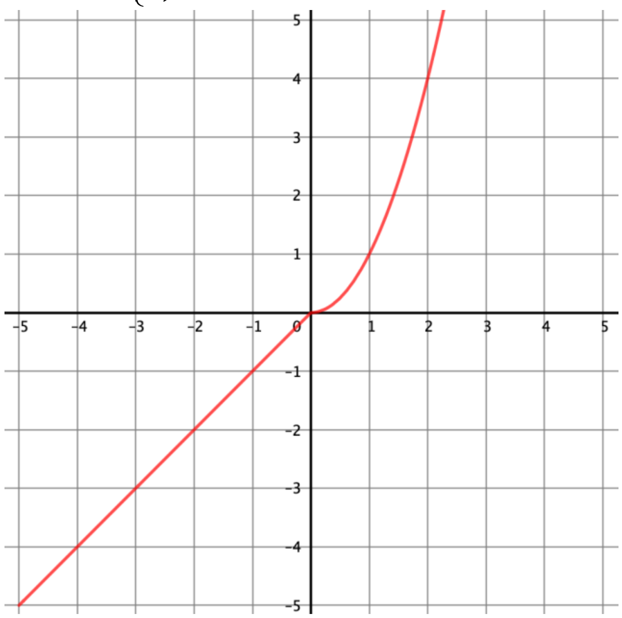
2.) Sketch a function having the following attributes, if possible.

a.) differentiable and continuous at the point (2,4) 	**b.)** continuous, but not differentiable at (-3,1) 	**c.)** cusp at the point (-1, 3) 	**d.)** differentiable, but not continuous at (2,-4) not possible. Differentiable then continuous. 

Skill Builder: Topics 2.4 – Differentiability

3.) For each function, *f*(*x*), determine if the function is continuous or non-continuous, differentiable or non-differentiable, and sketch the curve.

a.) $f(x)=\begin{cases} x^2, & x\geq 0 \\ x, & x<0 \end{cases}$



$\lim_{x\to 0^-} f(x)=0 \qquad \lim_{x\to 0^+} f(x)=0 \qquad f(0)=0$

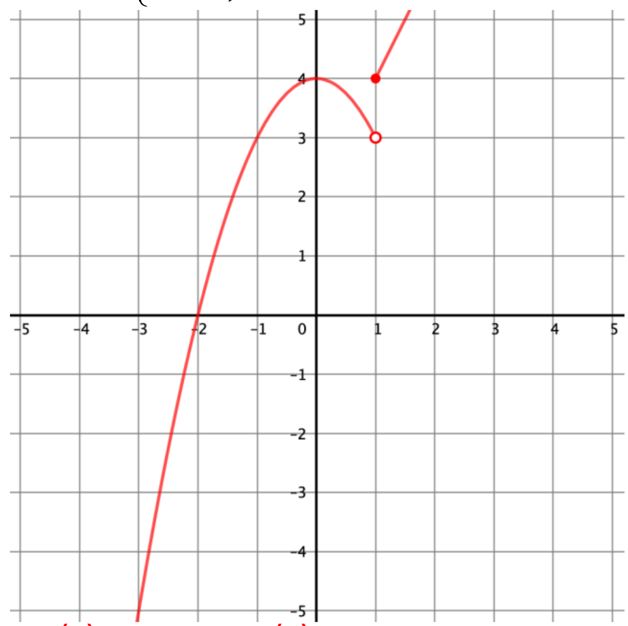
continuous

$\lim_{x\to 0^-} f'(x)=1 \qquad \lim_{x\to 0^+} f'(x)=2(0)=0 \quad f'(0)=0$

not differentiable

continuous   **differentiable**   **both**   **neither**

b.) $f(x)=\begin{cases} 4-x^2, & x<1 \\ 2x+2, & x\geq 1 \end{cases}$



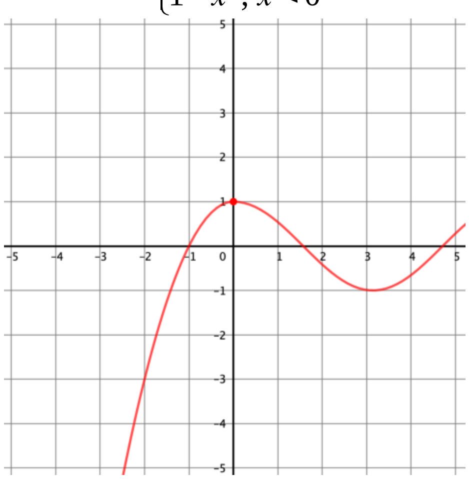
$\lim_{x\to 1^-} f(x)=3 \qquad \lim_{x\to 1^+} f(x)=4$

not continuous $\Rightarrow$ not differentiable

continuous   **differentiable**   **both**   **neither**

c.) $f(x)=\begin{cases} \cos x, & x\geq 0 \\ 1-x^2, & x<0 \end{cases}$

Note: Exercise caution when graphing $f(x)=\cos x$ using the provided coordinate axes.



$\lim_{x\to 0^-} f(x)=1 \qquad \lim_{x\to 0^+} f(x)=1 \qquad f(0)=1$

continuous

$\lim_{x\to 0^-} f'(x)=-2(0)=0 \qquad \lim_{x\to 0^+} f'(x)=\sin(0)=0$

$f'(0)=\sin(0)=0$ differentiable

continuous   **differentiable**   **both**   **neither**