Avon High School **Name** ______________________________

AP Calculus AB

Period _____ **Score** ______ **/ 10**

Skill Builder: Topics 2.8 & 2.9 – Product & Quotient Rules (Circuit)

Begin in the first cell marked #1 and find the derivative of each given function. To advance in the circuit, search for your answer and mark that cell #2. Continue in this manner until you complete the circuit. Show all pertinent work. Note: Some of these problems may require some algebraic manipulation either before or after you take the derivative.

_#1_ **Ans:** $-\dfrac{7}{x^2}$

$y=(2x-5)(3x+7)$

$y'=2(3x+7)+(2x-5)(3)$

$y'=6x+14+6x-15$

$y'=12x-1$

_16_ **Ans:** $\dfrac{5x^2+18x+9}{2\sqrt{x}}$

$y=\cos^2 x$

$y=\cos x\cdot\cos x$

$y'=(-\sin x)(\cos x)+(\cos x)(-\sin x)$

$y'=-2\sin x\cos x$

_9_ **Ans:** $\sec^2 x$

$y=\dfrac{3x+7}{x^3}$

$y'=\dfrac{(3)(x^3)-(3x+7)(3x^2)}{\left(x^3\right)^2}$

$y'=\dfrac{3x^3-9x^3-21x^2}{x^6}$

$y'=\dfrac{-6x^3-21x^2}{x^6}$

$y'=\dfrac{-3x^2(2x+7)}{x^6}$

$y'=\dfrac{-3(2x+7)}{x^4}$

_6_ **Ans:** $-\dfrac{9}{(3x+7)^2}$

$y=\dfrac{1+\cos x}{1+\sin x}$

$y'=\dfrac{-\sin x(1+\sin x)-(1+\cos x)(\cos x)}{(1+\sin x)^2}$

$y'=\dfrac{-\sin x-\sin^2 x-\cos x-\cos^2 x}{(1+\sin x)^2}$

$y'=\dfrac{-\left(\sin x+\sin^2 x+\cos x+\cos^2 x\right)}{(1+\sin x)^2}$

$y'=\dfrac{-(\sin x+\cos x+1)}{(1+\sin x)^2}$

_19_ **Ans:** $\dfrac{-2x\sin x-\cos x+x^2\cos x}{\sin^2 x}$

$y=\dfrac{x^2}{3x+7}$

_15_ **Ans:** $-\dfrac{2}{3}x^2-\dfrac{11}{9}x+\dfrac{17}{18}$

$y=\sqrt{x}\left(x+3\right)^2$

$$y' = \frac{2x(3x+7)-(x^2)(3)}{(3x+7)^2}$$
$$y' = \frac{6x^2+14x-3x^2}{(3x+7)^2}$$
$$y' = \frac{3x^2+14x}{(3x+7)^2}$$

$$y = x^{1/2}(x^2+6x+9)$$
$$y' = \frac{1}{2}x^{-1/2}\left(x^2+6x+9\right)+x^{1/2}(2x+6)$$
$$y' = \frac{1}{2}x^{3/2}+3x^{1/2}+\frac{9}{2}x^{-1/2}+2x^{3/2}+6x^{1/2}$$
$$y' = \frac{5}{2}x^{3/2}+9x^{1/2}+\frac{9}{2}x^{-1/2}$$
$$y' = \frac{5x^2+18x+9}{2\sqrt{x}}$$

_2_ **Ans:** $12x-1$

$$y = \frac{2x-5}{3x+7}$$
$$y' = \frac{2(3x+7)-(2x-5)(3)}{(3x+7)^2}$$
$$y' = \frac{6x+14-6x+15}{(3x+7)^2}$$
$$y' = \frac{29}{(3x+7)^2}$$

_12_ **Ans:** $\sec x \tan x$

Given $f(5)=3,\ f'(5)=3x+7,$

$g(5)=2x-1,\ g'(5)=\frac{1}{2}.$

If $h(t)=f(t)\cdot g(t)$, find $h'(5)$.

$$h'(t)=f'(t)\cdot g(t)+f(t)\cdot g'(t)$$
$$h'(5)=f'(5)\cdot g(5)+f(5)\cdot g'(5)$$
$$h'(5)=(3x+7)(2x-1)+3\left(\frac{1}{2}\right)$$
$$h'(5)=6x^2+11x-7+\frac{3}{2}=6x^2+11x-\frac{11}{2}$$

_10_ **Ans:** $\frac{-3(2x+7)}{x^4}$

$$y = \frac{3x+7}{\sqrt{x}}$$
$$y = \frac{3x+7}{x^{1/2}}$$
$$y' = \frac{3(x^{1/2})-(3x+7)\cdot\frac{1}{2}x^{-1/2}}{\left(x^{1/2}\right)^2}$$
$$y' = \frac{3x^{1/2}-\frac{3}{2}x^{1/2}-\frac{7}{2x^{1/2}}}{x}$$
$$y' = \frac{\frac{6x-3x-7}{2x^{1/2}}}{x}$$
$$y' = \frac{3x-7}{2x^{1/2}}\cdot\frac{1}{x}$$
$$y' = \frac{3x-7}{2x^{3/2}} \text{ or } \frac{3x-7}{2x\sqrt{x}}$$

_8_ **Ans:** $2\sin x\cos x$

$y=\tan x$ $\left(\text{Hint: Rewrite as } \frac{\sin x}{\cos x}\right)$

$$y = \frac{\sin x}{\cos x}$$
$$y' = \frac{(\cos x)(\cos x)-(\sin x)(-\sin x)}{\cos^2 x}$$
$$y' = \frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$
$$y' = \frac{1}{\cos^2 x}$$
$$y' = \sec^2 x$$

_4_ **Ans:** $x(2\cos x - x\sin x)$ $y = x\sin x$ $y' = 1\cdot(\sin x) + x(\cos x)$ $y' = \sin x + x\cos x$	**_17_** **Ans:** $-2\sin x\cos x$ $y = \csc x \quad \left(\text{Hint: Rewrite as } \dfrac{1}{\sin x}\right)$ $y = \dfrac{1}{\sin x}$ $y' = \dfrac{0(\sin x) - 1(\cos x)}{\sin^2 x}$ $y' = \dfrac{-\cos x}{\sin^2 x} = -\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin x}$ $y' = -\csc x \cdot \cot x$
_13_ **Ans:** $6x^2 + 11x - \dfrac{11}{2}$ $y = \cot x \quad \left(\text{Hint: Rewrite as } \dfrac{\cos x}{\sin x}\right)$ $y = \dfrac{\cos x}{\sin x}$ $y' = \dfrac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x}$ $y' = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ $y' = \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ $y' = \dfrac{-1}{\sin^2 x} = -\csc^2 x$	**_11_** **Ans:** $\dfrac{3x-7}{2x\sqrt{x}}$ $y = \sec x \quad \left(\text{Hint: Rewrite as } \dfrac{1}{\cos x}\right)$ $y = \dfrac{1}{\cos x}$ $y' = \dfrac{(0)(\cos x) - (1)(-\sin x)}{\cos^2 x}$ $y' = \dfrac{\sin x}{\cos^2 x}$ $y' = \dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x}$ $y' = \sec x \cdot \tan x$
_20_ **Ans:** $\dfrac{3x^2 + 14x}{(3x+7)^2}$ $y = \dfrac{3x^2 + 7x}{x^2}$ $y' = \dfrac{(6x+7)x^2 - (3x^2+7x)(2x)}{(x^2)^2}$ $y' = \dfrac{6x^3 + 7x^2 - 6x^3 - 14x^2}{x^4}$ $y' = \dfrac{-7x^2}{x^4} = -\dfrac{7}{x^2}$	**_18_** **Ans:** $-\csc x \cot x$ $y = \dfrac{1 - x^2}{\sin x}$ $y' = \dfrac{(-2x)(\sin x) - (1-x^2)(\cos x)}{\sin^2 x}$ $y' = \dfrac{-2x\sin x - \cos x + x^2\cos x}{\sin^2 x}$

_5_ **Ans:** $x\cos x+\sin x$ $y=\frac{3}{3x+7}$ $y'=\frac{0(3x+7)-(3)(3)}{(3x+7)^2}$ $y'=\frac{-9}{(3x+7)^2}$	**_3_** **Ans:** $\frac{29}{(3x+7)^2}$ $y=x^2\cos x$ $y'=2x(\cos x)+x^2(-\sin x)$ $y'=2x\cos x-x^2\sin x$ $y'=x(2\cos x-x\sin x)$
_14_ **Ans:** $-\csc^2 x$ Given $f(5)=3,\ f'(5)=3x+7,$ $g(5)=2x-1,\ g'(5)=\frac{1}{2}.$ If $k(t)=\frac{g(t)}{f(t)}$, find $k'(5)$. $k'(t)=\frac{g'(t)\cdot f(t)-g(t)\cdot f'(t)}{[f(t)]^2}$ $k'(5)=\frac{g'(5)\cdot f(5)-g(5)\cdot f'(5)}{[f(5)]^2}$ $k'(5)=\frac{\left(\frac{1}{2}\right)(3)-(2x-1)(3x+7)}{3^2}$ $k'(5)=\frac{\frac{3}{2}-6x^2-11x+7}{9}$ $k'(5)=\frac{3-12x^2-22x+14}{18}$ $k'(5)=\frac{-12x^2-22x+17}{18}=\frac{-2}{3}x^2-\frac{11}{9}x+\frac{17}{18}$	**_7_** **Ans:** $-\frac{\sin x+\cos x+1}{(1+\sin x)^2}$ $y=\sin^2 x$ (Hint: Rewrite as $\sin x\cdot\sin x$) $y=\sin x\cdot\sin x$ $y'=(\cos x)(\sin x)+(\sin x)(\cos x)$ $y'=2\sin x\cos x$

Find an equation of the tangent line to the graph of *f* at the indicated point and then use your calculator to confirm the results.

21.) $f(x)=\dfrac{x^2}{x-1}$ at $(2,4)$ $f'(x)=\dfrac{2x(x-1)-x^2(1)}{(x-1)^2}$ $f'(2)=\dfrac{2(2)(2-1)-(2)^2(1)}{(2-1)^2}$ $f'(2)=\dfrac{4-4}{1}$ $f'(2)=0$	**22.)** $f(x)=(x-2)(x^2-3x-1)$ at $(-1,-9)$ $f'(x)=(1)(x^2-3x-1)+(x-2)(2x-3)$ $f'(-1)=(1)(1+3-1)+(-3)(-5)$ $f'(-1)=3+15=18$

Determine the point(s) at which the graph of the following function as a horizontal tangent.

23.) $f(x)=\dfrac{x^2}{x^2-4}$ $f'(x)=\dfrac{2x(x^2-4)-x^2(2x)}{(x^2-4)^2}$ $f'(x)=\dfrac{2x^3-8x-2x^3}{(x^2-4)^2}$ $f'(x)=\dfrac{-8x}{(x^2-4)^2}$ $-8x=0$ $x=0$ $(0,0)$	**24.)** $f(x)=\dfrac{4x}{x^2+4}$ $f'(x)=\dfrac{4(x^2+4)-4x(2x)}{(x^2+4)^2}$ $f'(x)=\dfrac{4x^2+16-8x^2}{(x^2+4)^2}$ $f'(x)=\dfrac{-4x^2+16}{(x^2+4)^2}$ $-4x^2+16=0$ $4x^2=16$ $x^2=4$ $x=\pm 2$ $(2,1)$ and $(-2,-1)$

Use the chart to the right to find $h'(4)$ for Problems #25-30.

$f(4)$	$f'(4)$	$g(4)$	$g'(4)$
-8	3	3π	4

25.) $h(x)=5f(x)-\frac{2}{3}g(x)$

$h'(x)=5f'(x)-\frac{2}{3}g'(x)$

$h'(4)=5f'(4)-\frac{2}{3}g'(4)$

$h'(4)=5(3)-\frac{2}{3}(4)$

$h'(4)=15-\frac{8}{3}=\frac{37}{3}$

26.) $h(x)=3+8f(x)$

$h'(x)=0+8f'(x)$

$h'(4)=8f'(4)$

$h'(4)=8(3)$

$h'(4)=24$

27.) $h(x)=f(x)g(x)$

$h'(x)=f'(x)\cdot g(x)-f(x)\cdot g'(x)$

$h'(4)=f'(4)\cdot g(4)-f(4)\cdot g'(4)$

$h'(4)=(3)(3\pi)+(-8)(4)$

$h'(4)=9\pi-32$

28.) $h(x)=\frac{f(x)}{g(x)}$

$h'(x)=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{[g(x)]^2}$

$h'(4)=\frac{f'(4)\cdot g(4)-f(4)\cdot g'(4)}{[g(4)]^2}$

$h'(4)=\frac{(3)(3\pi)-(-8)(4)}{(3\pi)^2}$

$h'(4)=\frac{9\pi+32}{9\pi^2}$

29.) $h(x)=\frac{g(x)}{f(x)}$

$h'(x)=\frac{g'(x)\cdot f(x)-g(x)\cdot f'(x)}{[f(x)]^2}$

$h'(4)=\frac{g'(4)\cdot f(4)-g(4)\cdot f'(4)}{[f(4)]^2}$

$h'(4)=\frac{(4)(-8)-(3\pi)(3)}{(-8)^2}$

$h'(4)=\frac{-32-9\pi}{64}$

30.) $h(x)=\frac{f(x)+2}{-3g(x)}$

$h'(x)=\frac{(f'(x)+0)(-3g(x))-(f(x)+2)(-3g'(x))}{(-3g(x))^2}$

$h'(4)=\frac{f'(4)(-3g(4))-(f(4)+2)(-3g'(4))}{(-3g(4))^2}$

$h'(4)=\frac{(3)(-3)(3\pi)-(-8+2)(-3)(4)}{(-3\cdot 3\pi)^2}$

$h'(4)=\frac{-27\pi-72}{81\pi^2}=\frac{-9(3\pi+8)}{81\pi^2}=-\frac{3\pi+8}{9\pi^2}$