Topic 5.3 – Determining an Interval on Which a Function is Increasing or Decreasing

Topic 5.4 – Using the First Derivative Test to Determine Relative (Local) Extrema

Find the intervals where the function is increasing or decreasing. Use a sign chart to organize your analysis.

1.) $f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

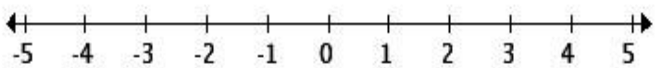
$f'(x) = 0$ $\qquad$ $f'(x)$ is und

$3x^2 - 3 = 0$ $\qquad$ $\varnothing$

$3(x^2 - 1) = 0$

$x = -1, 1$

Sign of $f'(x)$ $\quad$ + + + + + + + 0 - - - - 0 + + + + + + + + + +



$f(x)$ is increasing on $(-\infty, -1]$ and $[1, \infty)$ because $f'(x) > 0$ on those intervals.

$f(x)$ is decreasing on $[-1, 1]$ because $f'(x) < 0$ on that interval.

2.) $f(x) = x^4 - 8x^2 + 1$

$f'(x) = 4x^3 - 16x$

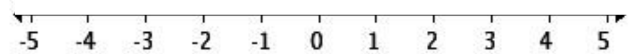
$f'(x) = 0$ $\qquad$ $f'(x)$ is und

$4x^3 - 16x = 0$ $\qquad$ $\varnothing$

$4x(x^2 - 4) = 0$

$x = -2, 0, 2$

Sign of $f'(x)$ $\quad$ - - - - - - 0 + + + 0 - - - 0 + + + + + +



$f(x)$ is decreasing on $(-\infty, -2]$ and $[0, 2]$ because $f'(x) < 0$ on those intervals.

$f(x)$ is increasing on $[-2, 0]$ and $[2, \infty)$ because $f'(x) > 0$ on those intervals.

3.) $f(x) = (x+1)^{2/3}$

$f'(x) = \frac{2}{3}(x+1)^{-1/3} = \frac{2}{3\sqrt[3]{x+1}}$

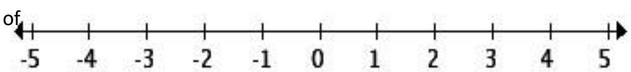
$f'(x) = 0$ $\qquad$ $f'(x)$ is und

$\varnothing$ $\qquad$ $3\sqrt[3]{x+1} = 0$

$x + 1 = 0$

$x = -1$

Sign of $f'(x)$ $\quad$ - - - - - - - - - - - 0 + + + + + + + + + + + + +



$f(x)$ is decreasing on $(-\infty, -1]$ because $f'(x) < 0$ on that interval.

$f(x)$ is increasing on $[-1, \infty)$ because $f'(x) > 0$ on that interval.

4.) $f(x) = \sin x + \cos x$ on $(0, 2\pi)$

$f'(x) = \cos x - \sin x$

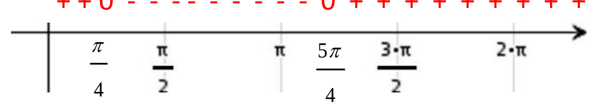
$f'(x) = 0$ $\qquad$ $f'(x)$ is und

$\cos x - \sin x = 0$ $\qquad$ $\varnothing$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Sign of $f'(x)$ $\quad$ + + 0 - - - - - - - - - - 0 + + + + + + + + +



$f(x)$ is increasing on $\left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$ because $f'(x) > 0$ on those intervals.

$f(x)$ is increasing on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ because $f'(x) < 0$ on that interval.

Find all critical numbers and use the First Derivative Test to find the points that are a relative maximum or a relative minimum. You may use a chart or a number line to perform your sign tests.

5.) $f(x) = x^4 + 4x^3 - 2$

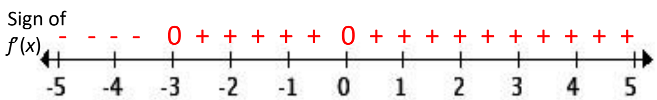
$f'(x) = 4x^3 + 12x^2$

$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

$4x^3 + 12x^2 = 0$ $\qquad$ $\varnothing$

$4x^2(x+3) = 0$

$x = -3,\ 0$



$f(x)$ has a relative minimum at $(-3,-29)$ because the sign of $f'(x)$ changes from negative to positive at $x = -3$.

6.) $f(x) = x^2 - 2x^{3/2} + 2$

$f'(x) = 2x - 3x^{1/2}$

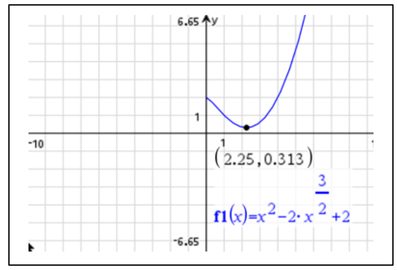
$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

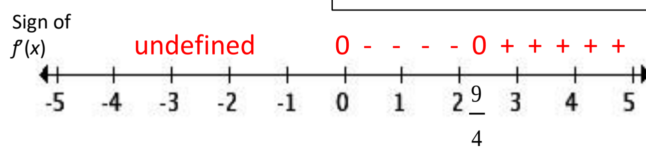
$2x - 3x^{1/2} = 0$ $\qquad$ $x < 0$

$x^{1/2}(2x^{1/2} - 3) = 0$

$x^{1/2} = 0, \quad 2x^{1/2} - 3 = 0$

$x = 0, \quad x = \dfrac{9}{4}$





$f(x)$ has a relative minimum at $\left(\dfrac{9}{4}, \dfrac{5}{16}\right)$ because the sign of $f'(x)$ changes from negative to positive at $x = \dfrac{9}{4}$.

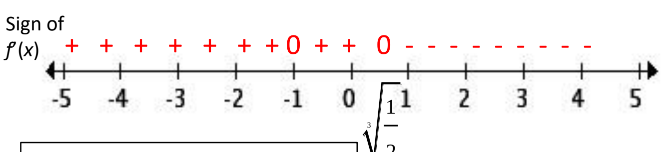
7.) $f(x) = \dfrac{x}{1+x^3}$

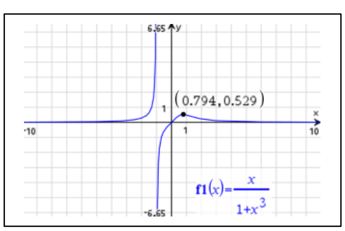
$f'(x) = \dfrac{1(1+x^3) - x(3x^2)}{(1+x^3)^2} = \dfrac{1-2x^3}{(1+x^3)^2}$

$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

$1 - 2x^3 = 0$ $\qquad$ $(1+x^3)^2 = 0$

$x = \sqrt[3]{\dfrac{1}{2}}$ $\qquad$ $x = -1$





$f(x)$ has a relative maximum at $\left(\dfrac{1}{\sqrt[3]{2}}, \dfrac{2}{3\sqrt[3]{2}}\right)$ because the sign of $f'(x)$ changes from positive to negative at $x = \sqrt[3]{\dfrac{1}{2}}$.

8.) $f(x) = \sqrt{x^3 + 3x^2}$ $\qquad$ Domain is $-3 \le x < \infty$

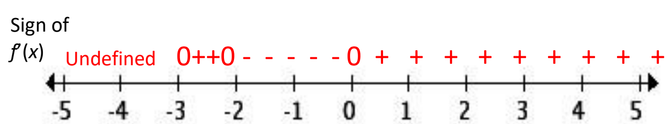
$f'(x) = \dfrac{1}{2}(x^3 + 3x^2)^{-1/2}(3x^2 + 6x) = \dfrac{3x^2 + 6x}{2\sqrt{x^3 + 3x^2}}$

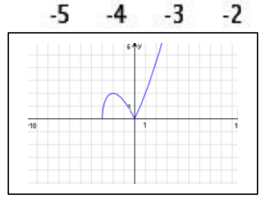
$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

$3x^2 + 6x = 0$ $\qquad$ $2\sqrt{x^3 + 3x^2} = 0$

$3x(x+2) = 0$ $\qquad$ $2\sqrt{x^2}\sqrt{x+3} = 0$

$x = -2,\ 0$ $\qquad$ $x = -3,\ 0$

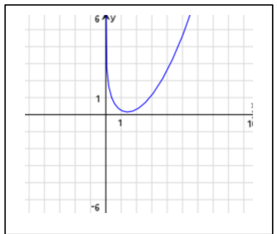




$f(x)$ has a relative maximum at $(-2, 2)$ because the sign of $f'(x)$ changes from positive to negative at $x = 2$.

$f(x)$ has a relative minimum at $(0, 0)$ because the sign of $f'(x)$ changes from neg to pos at $x = 0$.

9.) $y=\frac{x^2}{4}-\ln x$ Domain is $0<x<\infty$



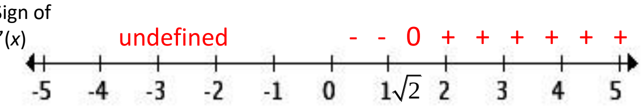
$f'(x)=\frac{1}{2}x-\frac{1}{x}=\frac{x^2-2}{2x}$

$\underline{f'(x)=0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

$x^2-2=0$ $\qquad$ $2x=0$

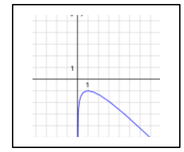
$x=\cancel{-\sqrt{2}},\ \sqrt{2}$ $\qquad$ $x=0$ (not in domain of $f(x)$)

Sign of $f'(x)$



$f(x)$ has a relative minimum at $\left(\sqrt{2},\frac{1}{2}-\ln\sqrt{2}\right)$ because the sign of $f'(x)$ changes from negative to positive at $x=\sqrt{2}$.

10.) $y=\ln x-x$ Domain is $0<x<\infty$



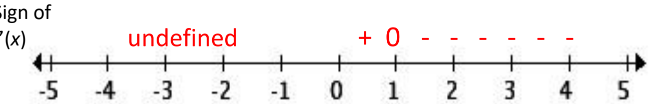
$f'(x)=\frac{1}{x}-1=\frac{1-x}{x}$

$\underline{f'(x)=0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

$1-x=0$ $\qquad$ $x=0$ (not in domain of $f(x)$)

$x=1$

Sign of $f'(x)$



$f(x)$ has a relative maximum at $(1,-1)$ because the sign of $f'(x)$ changes from positive to negative at $x=1$.

11.) $y=2x^2\ln x$ Domain is $0<x<\infty$

$f'(x)=4x\cdot\ln x+2x^2\cdot\frac{1}{x}=4x\cdot\ln x+2x$

$\underline{f'(x)=0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

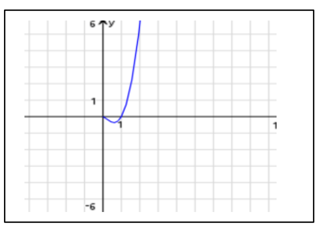
$4x\cdot\ln x+2x=0$ $\qquad$ $x\le 0$

$2x(2\ln x+1)=0$

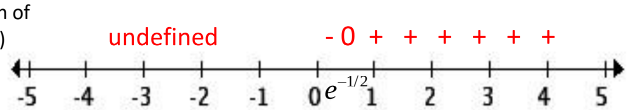
$2x=0 \quad 2\ln x+1=0$

$x=0 \quad \ln x=-\frac{1}{2}$

$x=e^{-1/2}$



Sign of $f'(x)$



$f(x)$ has a relative minimum at $\left(e^{-1/2},-e\right)$ because the sign of $f'(x)$ changes from negative to positive at $x=e^{-1/2}$.

Note: $e^{-1/2}$ is approximately 0.6. This is likely not common knowledge and you shouldn't worry too much if you did not know it. As far as selecting a test value between 0 and $e^{-1/2}$, e^{-1} would work rather easily.

12.) $y=\frac{\ln x}{2x}$ Domain is $0<x<\infty$

$f'(x)=\frac{\frac{1}{x}\cdot 2x-\ln x\cdot 2}{4x^2}\cdot\frac{x}{x}=\frac{2x-2x\ln x}{4x^3}$

$\underline{f'(x)=0}$ $\qquad$ $\underline{f'(x) \text{ is und}}$

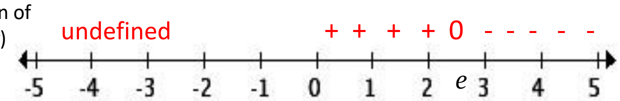
$2x-2x\ln x=0$ $\qquad$ $4x^3=0$

$2x(1-\ln x)=0$ $\qquad$ $x=0$ (not in domain of $f(x)$)

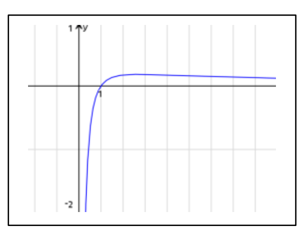
$2x=0 \quad \ln x=1$

$x=0 \quad x=e$

Sign of $f'(x)$



$f(x)$ has a relative maximum at $\left(e,\frac{1}{2e}\right)$ because the sign of $f'(x)$ changes from positive to negative at $x=e$.



13.) $y = 2xe^{-x}$

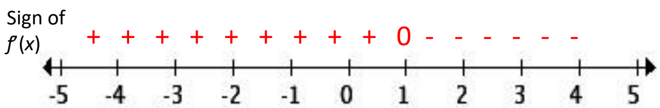
$f'(x) = 2 \cdot e^{-x} + 2x \cdot \left(-e^{-x}\right) = 2e^{-x}(1-x)$

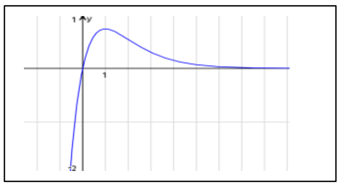
$\underline{f'(x)=0}$ $\underline{f'(x) \text{ is und}}$

$2e^{-x}(1-x)=0$ $\varnothing$

$2e^{-x}=0$ $1-x=0$

$\varnothing$ $x=1$





$f(x)$ has a relative maximum at $\left(1, \frac{2}{e}\right)$ because the sign of $f'(x)$ changes from positive to negative at $x=1$.

14.) $y = x^2e^{-x}$

$f'(x) = 2x \cdot e^{-x} + x^2 \cdot \left(-e^{-x}\right) = 2xe^{-x} - x^2e^{-x}$

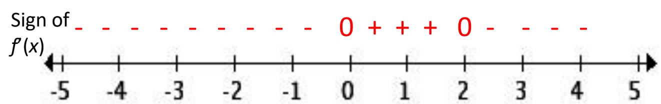
$\underline{f'(x)=0}$ $\underline{f'(x) \text{ is und}}$

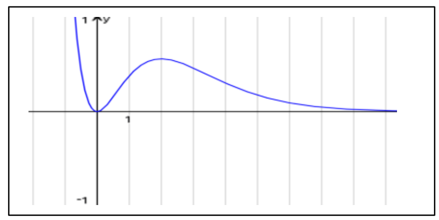
$2xe^{-x} - x^2e^{-x} = 0$ $\varnothing$

$xe^{-x}(2-x)=0$

$x=0$ $e^{-x}=0$ $2-x=0$

$x=0$ $\varnothing$ $x=2$





$f(x)$ has a relative minimum at $(0,0)$ because the sign of $f'(x)$ changes from negative to positive at $x=0$.

$f(x)$ has a relative maximum at $\left(2, \frac{4}{e^2}\right)$ because the sign of $f'(x)$ changes from positive to negative at $x=2$.