AP Calculus AB SOLUTIONS

Skill Builder: Topic 7.2 – Verifying Solutions to Differential Equations

1. A curve has slope $2x+3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y=5x+3$ **(B)** $y=x^2+1$ **(C)** $y=x^2+3x$ **(D)** $y=x^2+3x-2$

slope is $2x+3$ implies $y'=2x+3$

(A) $y'=5$ Eliminates Choice (A) (C) $y'=2x+3$ Choice (C) is a possibility

(B) $y'=2x$ Eliminates Choice (B) (D) $y'=2x+3$ Choice (D) is a possibility

To determine which of Choice C or Choice D is correct, the equation must contain the point $(1,2)$. Only Choice (D) meet that requirement.

2. Which of the following is the solution to the differential equation $\frac{dy}{dx}=e^{y+x}$ with the initial condition $y(0)=-\ln 4$?

(A) $y=-x-\ln 4$ **(B)** $y=x-\ln 4$ **(C)** $y=-\ln\left(-e^x+5\right)$

(D) $y=-\ln\left(e^x+3\right)$ **(E)** $y=\ln\left(e^x+3\right)$

(A) $\frac{dy}{dx}=-1$ So, $-1=e^{-x-\ln 4+x}=e^{-\ln(4)}=e^{\ln 4^{-1}}=4^{-1}=\frac{1}{4}$ Not true.

(B) $\frac{dy}{dx}=1$ So, $1=e^{x-\ln 4+x}=e^{2x-\ln(4)}=\frac{e^{2x}}{e^{\ln(4)}}=\frac{e^{2x}}{4}$ Not true.

(C) $\frac{dy}{dx}=\frac{e^x}{-e^x+5}$ So, $\frac{e^x}{-e^x+5}=e^{-\ln(-e^x+5)+x}=e^{x-\ln(-e^x+5)}=\frac{e^x}{e^{\ln(-e^x+5)}}=\frac{e^x}{-e^x+5}$ True.

(D) $\frac{dy}{dx}=-\frac{e^x}{e^x+3}$ So, $-\frac{e^x}{e^x+3}=e^{\ln(e^x+3)+x}=e^{\ln(e^x+3)}\cdot e^x=(e^x+3)\cdot e^x$ Not true.

(E) $\frac{dy}{dx}=\frac{e^x}{e^x+3}$ So, $\frac{e^x}{e^x+3}=e^{\ln(e^x+3)+x}=e^{\ln(e^x+3)}\cdot e^x=(e^x+3)\cdot e^x$ Not true.

3. If $\dfrac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ **(B)** $e^{\tan x} + 5$ **(C)** $5e^{\tan x}$

(D) $\tan x + 5$ **(E)** $\tan x + 5e^x$

(A) $\dfrac{dy}{dx} = \sec^2 x \cdot e^{\tan x}$ So, $\sec^2 x \cdot e^{\tan x} = \left(e^{\tan x} + 4\right)\sec^2 x$ Not true.

(B) $\dfrac{dy}{dx} = \sec^2 x \cdot e^{\tan x}$ So, $\sec^2 x \cdot e^{\tan x} = \left(e^{\tan x} + 5\right)\sec^2 x$ Not true.

(C) $\dfrac{dy}{dx} = 5\sec^2 x \cdot e^{\tan x}$ So, $5\sec^2 x \cdot e^{\tan x} = \left(5e^{\tan x}\right)\sec^2 x$ True.

(D) $\dfrac{dy}{dx} = \sec^2 x$ So, $\sec^2 x = \left(\sec^2 x\right)\sec^2 x$ Not true.

(E) $\dfrac{dy}{dx} = \sec^2 x + 5e^x$ So, $\sec^2 x + 5e^x = \left(\tan x + 5e^x\right)\sec^2 x$ Not true.

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4. Which of the following is a solution to the differential equation $y'' - 4y = 0$? Circle all that apply.

(A) $y = e^{2x}$ **(B)** $y = 2e^{2x}$ **(C)** $y = \sin(2x)$ **(D)** $y = \cos(2x)$

(A) $y' = 2e^{2x}$; $y'' = 4e^{2x}$ So, $y'' - 4y = 4e^{2x} - 4e^{2x} = 0$ True.

(B) $y' = 4e^{2x}$; $y'' = 8e^{2x}$ So, $y'' - 4y = 8e^{2x} - 4\left(2e^{2x}\right) = 0$ True.

(C) $y' = 2\cos(2x)$; $y'' = -4\sin(2x)$ So, $y'' - 4y = -4\sin(2x) - 4\sin(2x) = 0$ Not true.

(D) $y' = -2\sin(2x)$; $y'' = -4\cos(2x)$ So, $y'' - 4y = -4\cos(2x) - 4\cos(2x) = 0$ Not true.

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5. Of the following, which are solutions to the differential equation $y'' - 10y' + 9y = 0$?

I. $y = 2\sin(3x)$ **II.** $y = 5e^x$ **III.** $y = Ce^{9x}$, where C is a constant.

(A) II only **(B)** III only **(C)** II and III only **(D)** I and III only **(E)** I, II, and III

I. $y' = 6\cos(3x)$; $y'' = -18\sin(3x)$ So, $-18\sin(3x) - 60\cos(3x) + 18\sin(3x) = 0$ Not true.

II. $y' = 5e^x$; $y'' = 5e^x$ So, $5e^x - 50e^x + 45e^x = 0$ True.

III. $y' = 9Ce^{9x}$; $y'' = 81Ce^{9x}$ So, $81Ce^{9x} - 90Ce^{9x} + 9Ce^{9x} = 0$ True.

6. For what value of k, if any, will $y = k\sin(5x) + 2\cos(4x)$ be a solution to the differential equation $y'' + 16y = -27\sin(5x)$?

(A) -27 **(B)** $-\frac{9}{5}$ **(C)** 3 **(D)** There is no such value of k.

$$y' = 5k\cos(5x) - 8\sin(4x)$$

$$y'' = -25k\sin(5x) - 32\cos(4x)$$

So, $-25k\sin(5x) - 32\cos(4x) + 16\left(k\sin(5x) + 2\cos(4x)\right) = -27\sin(5x)$

$$-25k\sin(5x) - 32\cos(4x) + 16k\sin(5x) + 32\cos(4x) = -27\sin(5x)$$

$$-9k\sin(5x) = -27\sin(5x)$$

$$-9k = -27$$

$$k = 3$$

7. For what value of k, if any, is $y = e^{-2x} + ke^{4x}$ be a solution to the differential equation $y - \frac{y''}{4} = 5e^{4x}$?

(A) $-\frac{5}{3}$ **(B)** $\frac{20}{3}$ **(C)** 5 **(D)** There is no such value of k.

$$y' = -2e^{-2x} + 4ke^{4x}$$

$$y'' = 4e^{-2x} + 16ke^{4x}$$

So, $e^{-2x} + ke^{4x} - \frac{4e^{-2x} + 16ke^{4x}}{4} = 5e^{4x}$

$$e^{-2x} + ke^{4x} - e^{-2x} - 4ke^{4x} = 5e^{4x}$$

$$-3ke^{4x} = 5e^{4x}$$

$$-3k = 5$$

$$k = -\frac{5}{3}$$

8. For what value of k, if any, will $y = ke^{-2x} + 4\cos(3x)$ be a solution to the differential equation $y'' + 9y = 26e^{-2x}$?

(A) 2   **(B)** $\frac{13}{5}$ **(C)** 26   **(D)** There is no such value of k.

$y' = -2ke^{-2x} - 12\sin(3x)$

$y'' = 4ke^{-2x} - 36\cos(3x)$

So, $4ke^{-2x} - 36\cos(3x) + 9\left(ke^{-2x} + 4\cos(3x)\right) = 26e^{-2x}$

$$4ke^{-2x} - 36\cos(3x) + 9ke^{-2x} + 36\cos(3x) = 26e^{-2x}$$

$$13ke^{4x} = 26e^{-2x}$$

$$13k = 26$$

$$k = 2$$