AP Calculus AB **SOLUTIONS**

Skill Builder: Topic 7.6 – Finding General Solutions Using Separation of Variables

Find a general solution to each of the following differentiable equations.
If it is possible to solve for y easily, then do so.

1. $\frac{dy}{dx} = \frac{x}{y}$	$y\,dy = x\,dx$ $\int y\,dy = \int x\,dx$ $\frac{y^2}{2} = \frac{x^2}{2} + C$ $y^2 = x^2 + C$ or $y = \pm\sqrt{x^2 + C}$	Notes on the problem While it was not too difficult to solve for y in our solution to the DEQ, leavnig the answer as $y^2 = x^2 + C$ or even $y^2 - x^2 + C$ could be preferred as it makes it easy to notice the family of solution curves are hyperbolas.
2. $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$	$3y^2\,dy = \left(x^2+2\right)dx$ $\int 3y^2\,dy = \int \left(x^2+2\right)dx$ $y^3 = \frac{x^3}{3} + 2x + C$	
3. $x\frac{dy}{dx} = y$	$\frac{1}{y}dy = \frac{1}{x}dx$ $\int \frac{1}{y}dy = \int \frac{1}{x}dx$ $\ln\lvert y\rvert = \ln\lvert x\rvert + C_0$ $e^{\ln\lvert y\rvert} = e^{\ln\lvert x\rvert + C_0} = e^{\ln\lvert x\rvert}\cdot e^{C_0}$ $y = C\cdot x$	Notes on the problem Because the domain of the original values of x and y are all real numbers except $x \neq 0, y \neq 0$, the absolute values can be removed so as to not impose any unrequired restrictions on our solution. This is common in the solutions to many DEQs.
4. $(2+x)\frac{dy}{dx} = 3y$	$\frac{1}{y}dy = \frac{3}{2+x}dx$ $\int \frac{1}{y}dy = \int \frac{3}{2+x}dx$ $\ln\lvert y\rvert = 3\ln\lvert 2+x\rvert + C_0$ $e^{\ln\lvert y\rvert} = e^{\ln\lvert 2+x\rvert^3 + C_0} = e^{\ln\lvert 2+x\rvert^3}\cdot e^{C_0}$ $y = C(x+2)^3$	

5. $yy' = \sin x$	$y\,dy = \sin x\,dx$ $\int y\,dy = \int \sin x\,dx$ $\frac{y^2}{2} = -\cos x + C$ $y^2 = -2\cos x + C$	
6. $(1+4x^2)y' = 1$	$dy = \frac{1}{1+4x^2}dx$ $\int dy = \int \frac{1}{1+4x^2}dx$ $y = \frac{1}{2}\arctan(2x) + C$	Notes on the problem The antiderivative of the right side uses the substitution $u = 2x \quad du = 2\,dx$ results in the form $\int \frac{1}{1+u^2}\frac{du}{2}$