AP Calculus AB **SOLUTIONS**

Skill Builder: Topic 7.7 – Finding Particular Solutions Using Separation of Variables

Find the solution of the differential equation that satisfies the given condition.

1. $\sqrt{x}+\sqrt{y}\cdot\dfrac{dy}{dx}=0,\ y(1)=4$

$\sqrt{y}dy=-\sqrt{x}\,dx$

$\int\sqrt{y}\,dy=-\int\sqrt{x}\,dx$

$\dfrac{2}{3}y^{3/2}=-\dfrac{2}{3}x^{3/2}+C$

Use $(1,4)$ to find C

$\dfrac{2}{3}(4)^{3/2}=-\dfrac{2}{3}(1)^{3/2}+C$

$\dfrac{16}{3}=-\dfrac{2}{3}+C$

$C=\dfrac{18}{3}$

So, $\dfrac{2}{3}y^{3/2}=-\dfrac{2}{3}x^{3/2}+\dfrac{18}{3}\ \rightarrow\ y^{3/2}=-x^{3/2}+9$

2. $y\dfrac{dy}{dx}=e^x,\ y(0)=4$

$ydy=e^x dx$

$\int y\,dy=\int e^x\,dx$

$\dfrac{1}{2}y^2=e^x+C$

Use $(0,4)$ to find C

$\dfrac{1}{2}(4)^2=e^0+C$

$8=1+C$

$C=7$

So, $\dfrac{1}{2}y^2=e^x+7\ \rightarrow\ y^2=2e^x+14$

3. $xy\dfrac{dy}{dx} - \ln x = 0, \quad y(1) = 0$

$$y\,dy = \frac{\ln x}{x}\,dx$$

$$\int y\,dy = \int \frac{\ln x}{x}\,dx$$

$$\frac{1}{2}y^2 = \frac{(\ln x)^2}{2} + C$$

Use $(1,0)$ to find C

$$\frac{1}{2}(0)^2 = \frac{(\ln 1)^2}{1} + C$$

$$0 = 0 + C$$

$$C = 0$$

So, $\dfrac{1}{2}y^2 = \dfrac{(\ln x)^2}{2} \rightarrow y^2 = (\ln x)^2$

4. $y(x+1) + \dfrac{dy}{dx} = 0, \quad y(-2) = 1$

$$\frac{dy}{dx} = -y(x+1)$$

$$-\frac{1}{y}\,dy = (x+1)\,dx$$

$$-\int \frac{1}{y}\,dy = \int (x+1)\,dx$$

$$-\ln|y| = \frac{x^2}{2} + x + C$$

Use $(-2,1)$ to find C

$$-\ln|1| = \frac{(-2)^2}{2} + (-2) + C$$

$$0 = 2 - 2 + C$$

$$C = 0$$

So, $-\ln|y| = \dfrac{x^2}{2} + x \rightarrow \ln|y| = -\dfrac{x^2}{2} - x$

$$e^{\ln|y|} = e^{-\frac{x^2}{2} - x}$$

$$y = e^{-\frac{x^2+2x}{2}}$$

5. $(1+x^2)\dfrac{dy}{dx}-(1+y^2)=0,\ \ y(0)=\sqrt{3}$

$$\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$$

$$\frac{1}{1+y^2}\,dy=\frac{1}{1+x^2}\,dx$$

$$\int\frac{1}{1+y^2}\,dy=\int\frac{1}{1+x^2}\,dx$$

$$\arctan y=\arctan x+C$$

Use $(0,\sqrt{3})$ to find C

$$\arctan\sqrt{3}=\arctan 0+C$$

$$\frac{\pi}{3}=0+C$$

$$C=\frac{\pi}{3}$$

So, $\arctan y=\arctan x+\dfrac{\pi}{3}$

6. $dT+k(T-70)dt=0,\ \ T(0)=140$

$$dT=-k(T-70)\,dt$$

$$\frac{1}{T-70}\,dT=-k\,dt$$

$$\int\frac{1}{T-70}\,dT=-\int k\,dt$$

$$\ln|T-70|=-kt+C$$

Use $(0,140)$ to find C

$$\ln|140-70|=-k(0)+C$$

$$C=\ln 70$$

So, $\ln|T-70|=-kt+\ln 70$

$$e^{\ln|T-70|}=e^{-kt+\ln 70}$$

$$e^{\ln|T-70|}=e^{-kt}\cdot e^{\ln 70}$$

$$T-70=70e^{-kt}$$

$$T=70+70e^{-kt}$$