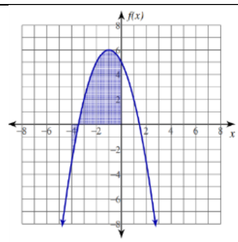
SOLUTIONS

AP Calculus AB
Skill Builder: 8.1 – Finding the Average Value of a Function on an Interval

Find the average value of the function on the given interval. The graphs are provided for you to help verify your answers.

1. $f(x) = -x^2 - 2x + 5 \quad [-4, 0]$



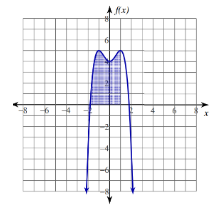
$$f_{\text{avg}} = \frac{1}{0-(-4)} \int_{-4}^{0} \left(-x^2 - 2x + 5\right) dx$$

$$= \frac{1}{4}\left(-\frac{x^3}{3} - x^2 + 5x\right)\Bigg|_{-4}^{0}$$

$$= \frac{1}{4}\left(0 - \left(\frac{64}{3} - 16 - 20\right)\right)$$

$$= \frac{1}{4}\left(\frac{44}{3}\right) = \frac{11}{3}$$

2. $f(x) = -x^4 + 2x^2 + 4 \quad [-2, 1]$



$$f_{\text{avg}} = \frac{1}{1-(-2)} \int_{-2}^{1} \left(-x^4 + 2x^2 + 4\right) dx$$

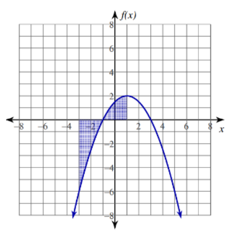
$$= \frac{1}{3}\left(-\frac{x^5}{5} + \frac{2x^3}{3} + 4x\right)\Bigg|_{-2}^{1}$$

$$= \frac{1}{3}\left(-\frac{1}{5} + \frac{2}{3} + 4 - \left(\frac{32}{5} - \frac{16}{3} - 8\right)\right)$$

$$= \frac{1}{3}\left(\frac{57}{5}\right) = \frac{19}{5}$$

Find the value(s) of *c* guaranteed by the Mean Value Theorem for Integrals for the function over the given interval. The graphs are provided for you to help verify your answers.

3. $f(x) = -\frac{x^2}{2} + x + \frac{3}{2} \quad [-3, 1]$



$$f_{\text{avg}} = \frac{1}{1-(-3)} \int_{-3}^{1} \left(-\frac{x^2}{2} + x + \frac{3}{2}\right) dx$$

$$= \frac{1}{4}\left(-\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{2}x\right)\Bigg|_{-3}^{1}$$

$$= \frac{1}{4}\left(-\frac{1}{6} + \frac{1}{2} + \frac{3}{2} - \left(\frac{27}{6} + \frac{9}{2} - \frac{9}{2}\right)\right)$$

$$= \frac{1}{4}\left(-\frac{8}{3}\right) = -\frac{2}{3}$$

$$-\frac{x^2}{2} + x + \frac{3}{2} = -\frac{2}{3}$$

$$-3x^2 + 6x + 9 = -4$$

$$3x^2 - 6x - 13 = 0$$

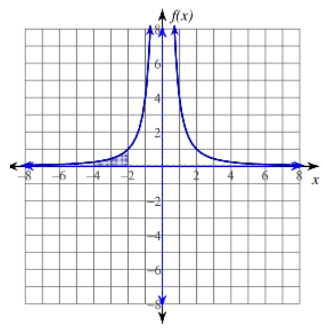
$$x = \frac{6 \pm \sqrt{36 - 4(3)(-13)}}{6}$$

$$x = \frac{6 \pm \sqrt{192}}{6}$$

$$x = \frac{6 \pm \sqrt{192}}{6} = \frac{6 \pm 8\sqrt{3}}{6} = \frac{3 \pm 4\sqrt{3}}{3}$$

Only $x = \frac{3 - 4\sqrt{3}}{3}$ lies within the interval $[-3, 1]$.

4. $f(x) = \frac{4}{x^2} \quad [-4, -2]$



$$f_{\text{avg}} = \frac{1}{-2-(-4)} \int_{-4}^{-2} \left(\frac{4}{x^2}\right) dx$$

$$= \frac{1}{2}\left(-\frac{4}{x}\right)\Bigg|_{-4}^{-2}$$

$$= \frac{1}{2}\left(\frac{-4}{-2} - \left(\frac{-4}{-4}\right)\right)$$

$$= \frac{1}{2}(2 - 1) = \frac{1}{2}$$

$$\frac{4}{x^2} = \frac{1}{2}$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

Only $x = -2\sqrt{2}$ lies within the interval $[-4, -2]$.

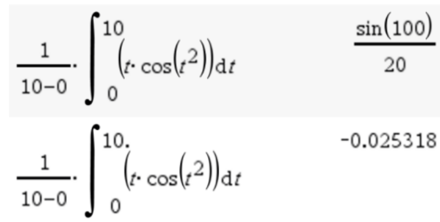
For Problem 5, use a calculator to evaluate. Be sure to write the expression you are entering into the calculator.

5. Compute the average value of the function $f(t) = t\cos t^2$ on the interval $[0,10]$.
First solve without using your calculator and then check your answer using your calculator.

$$f_{avg} = \frac{1}{10}\int_0^{10} t\cos t^2\,dt \qquad u = t^2 \quad du = 2tdt$$

$$= \frac{1}{10}\cdot\frac{1}{2}\int_0^{100}\cos u\,du = \frac{1}{20}\sin u\Big|_0^{100}$$

$$= \frac{1}{20}\sin 100$$



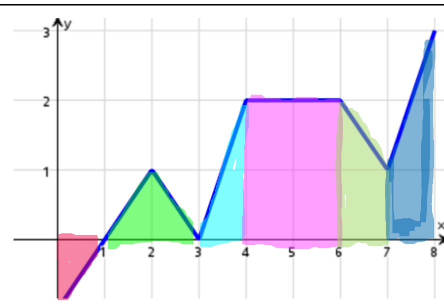
6. On a late winter day in Avon, Indiana, the temperature in $^\circ$F, t hours after 9 A.M. can be modeled by the function $T(t) = 55 + 14\cos\left(\frac{\pi t}{12}\right)$. Find the average temperature during the period from 9 A.M. to 9 P.M.

$$T_{\text{avg}} = \frac{1}{12-0}\int_0^{12}\left(55 + 14\cos\left(\frac{\pi t}{12}\right)\right)dt$$

$$= \frac{1}{12}\left(55t + \frac{12}{\pi}\sin\left(\frac{\pi t}{12}\right)\right)\Bigg|_0^{12} = \frac{1}{12}\left(660 + 0 - (0+0)\right) = \frac{660}{12} = 55^\circ\text{F}$$

Answer each of the following problems using the Average Value of a Function.

7. Find the average value of the function f, whose graph is shown to the right, on the interval $[0,8]$.



$$f_{\text{avg}} = \frac{1}{8-0}\int_0^8 f(x) = \frac{1}{8}\left(-\frac{1}{2} + 1 + 1 + 4 + \frac{3}{2} + 2\right)$$

$$= \frac{1}{8}\cdot 9$$

$$= \frac{9}{8}$$

8. Let $f(x)=\begin{cases} x^2 & \text{if } x\le 2 \\ 2x & \text{if } x>2 \end{cases}$. What is the average value of $f(x)$ on the interval $[1,4]$?

$$f_{\text{avg}}=\frac{1}{4-1}\left[\int_1^2 x^2\,dx+\int_2^4 2x\,dx\right]$$

$$=\frac{1}{3}\left[\left.\frac{x^3}{3}\right|_1^2+\left.x^2\right|_2^4\right]$$

$$=\frac{1}{3}\left(\frac{8}{3}-\frac{1}{3}+(16-4)\right)=\frac{1}{3}\left(\frac{7}{3}+12\right)=\frac{1}{3}\left(\frac{43}{3}\right)=\frac{43}{9}$$

9.

t **(minutes)**	0	4	8	12	16
***H*(*t*)** ($^\circ$C)	65	68	73	80	90

The temperature in degrees Celsius ($^\circ$C), of an oven being heated is modeled by an increasing differentiable function *H* of time *t*, where *t* is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

$$T_{\text{avg}}\approx\frac{1}{16-0}\left[(4)(65)+(4)(68)+(4)(73)+(4)(80)\right]$$

$$\approx\frac{1}{16}(1144)$$

$$\approx 71.5^\circ\text{C}$$

10. Find all the numbers *b* such that the average value of $f(x)=2+6x-3x^2$ on the interval $[0,b]$ is equal to 3.

$$f_{\text{avg}}=\frac{1}{b-0}\int_0^b\left(2+6x-3x^2\right)dx$$

$$=\frac{1}{b}\left.\left(2x+3x^2-x^3\right)\right|_0^b$$

$$=\frac{1}{b}\left(2b+3b^2-b^3-(0)\right)$$

$$=-b^2+3b+2$$

$$-b^2+3b+2=3$$

$$b^2-3b+1=0$$

$$b=\frac{3\pm\sqrt{9-4}}{2}$$

$$b=\frac{3\pm\sqrt{5}}{2}$$