AP Calculus AB **SOLUTIONS**

Skill Builder: 8.2 – Connecting Position, Velocity and Acceleration of Functions Using Integrals (Big 10)

Complete the ten problems below. Once you complete each problem, cross off the appropriate value in the box below according to the instructions. If done correctly, you will have all 10 numbers crossed off with no repeats.

0	1	2	3	4	5	6	7	8	9

A) A particle is moving along the *x*-axis for $t \geq 0$. The position of the particle is given by $x(t) = t^3 - 9t^2 - 21t + 6$. At what time, *t*, does the particle change directions?

The particle changes directions when the sign of its velocity, $v(t)$, changes.

$x'(t) = v(t) = 3t^2 - 18t - 21 = 0$

$3(t^2 - 6t - 7) = 0$ $\quad v(6) = 3(-1)(2) < 0$ $\quad v(8) = 3(1)(9) > 0$

$(t-7)(t+1) = 0$

$t = 7,\ \cancel{t = -1}$

7

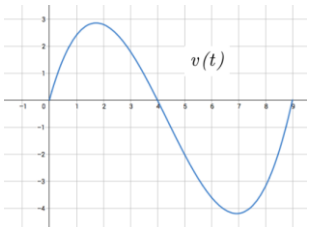
B) A bug is moving back and forth on a straight path. The velocity of the bug is given by $v(t) = t^2 - 3t$. Find the average acceleration of the bug on the interval $[1,4]$.

$$a_{avg}(t) = \frac{v(4) - v(1)}{4-1} = \frac{(4)^2 - 3(4) - \left((1)^2 - 3(1)\right)}{3} = \frac{16 - 12 - 1 + 3}{3} = \frac{6}{3} = 2$$

OR

$$a(t) = v'(t) = 2t - 3 \quad a_{avg}(t) = \frac{1}{4-1}\int_1^4 (2t-3)\,dt = \frac{1}{3}\left(t^2 - 3t\right)\Big|_1^4 = \frac{1}{3}\left(16 - 12 - (1-3)\right) = 2$$

2



C) The velocity of a particle for $0 \leq t \leq 9$ is given in the graph above. At which of the following values for *t* is the particle speeding up?

$t = 3$ $\quad t = 4$ $\quad t = 5$ $\quad t = 7$

The particle is speeding up (i.e. speed of the particle is increasing) when the sign of its velocity and acceleration are the same. At time $t = 5$, the velocity of the particle is negative (graph of *v* is below the *x*-axis) and the acceleration of the particle is negative (graph of *v* is decreasing). This is the only time when both $v(t)$ and $a(t)$ have the same sign. 5

D) The velocity of a particle is modeled by the equation $v(t)=\frac{1}{3}t^3-3t^2+7t+1$. What is the maximum acceleration of the particle on the interval $[0,8]$?

$a(t)=v'(t)=t^2-6t-7$

$a'(t)=2t-6=0$

$t=3$

Candidates: $0, 3,$ and 8

t	$a(t)$
0	$0^2-6(0)-7=-7$
3	$3^2-6(3)-7=-16$
8	$8^2-6(8)-7=9$

The maximum acceleration of the particle is 9 (which occurs at time $t = 8$).

9

E) A bug moves along a vertical post and its position is given by $y(t)$. The velocity of the bug is modeled by the equation $v(t)=2t-1$. When $t=1,\ y(t)=2$. Find $y(3)$.

$$y(3)=y(1)+\int_1^3 v(t)\,dt$$

$$=2+\int_1^3 (2t-1)\,dt=2+\left(t^2-t\right)\Big|_1^3=2+9-3-(1-1)=8$$

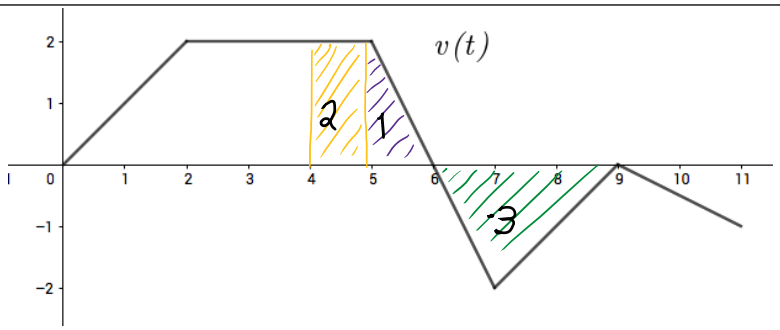
8

t	1	2	4	6	8
$x(t)$	-3	0	-1	4	5
$v(t)$	2	1	-3	3	0

F) A particle is moving along the *x*-axis. The position and velocity of the particle is recorded for various times in the table above. At which time, *t*, is the particle moving towards the origin?

In order for the particle to be moving toward the origin, one of two different scenarios must be occurring – i) the position of the particle is negative AND the velocity of the particle is positive (i.e. the particle moves to the right) or ii) the position of the particle is positive AND the velocity of the particle is negative (i.e. the particle moves to the left). At time $t = 1$, we see that case i) from above is true.

1



G) The velocity of a particle is given in the graph above for $0\le t\le 11$. What is the total distance traveled by the particle on the interval $[4,9]$?

Total distance traveled from $t=4$ to $t=9$ is

$$\int_4^9 |v(t)|\,dt=|2|+|1|+|-3|=6$$

6

H) The position of a bug moving along a straight path is given by $s(t) = t^2 - 2t + 3$. At what time, t, is the instantaneous velocity equal to the average velocity of the bug on $[0,6]$?

Instantaneous velocity at time t: $s'(t) = 2t - 2$

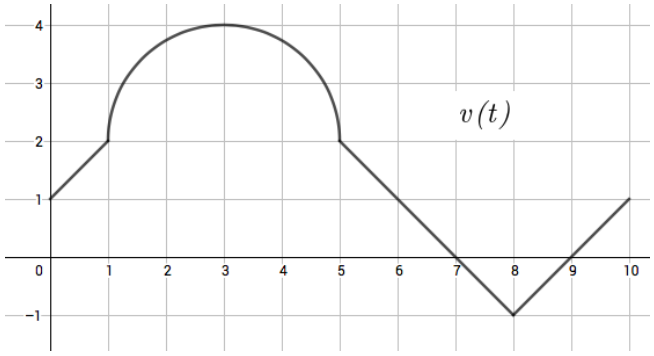
Average velocity on $[0,6]$: $\frac{s(6)-s(0)}{6-0} = \frac{36-12+3-(0-0+3)}{6} = \frac{24}{6} = 4$

$2t - 2 = 4$

$2t = 6$

$t = 3$

3



I) The velocity of an object is graphed above for $0 \le t \le 10$. What is the velocity of the object when the acceleration equals 0?

The acceleration of the object is 0 when the graph of velocity has a derivative of 0. This occurs at t = 3 in the graph above. At that time, the velocity is equal to 4.

4

J) The position of a bug moving along the x-axis is given by $x(t) = at + b$, where a and b are both nonzero. Find the acceleration of the bug at any time, t.

$v(t) = x'(t) = a$ Keep in mind that a is a constant.

$a(t) = v'(t) = 0$

0

Special thanks to Bryan Passwater, Speedway HS, Speedway, IN for authoring this activity.