Hw 1.16 Name___________________________

Intermediate Value Theorem

Skill Builder: Topic 1.16 – Working with the Intermediate Value Theorem

1. Let f be a function that is continuous on the closed interval [2,4] with $f(2)=10$ and $f(4)=20$. Which of the following is guaranteed by the Intermediate Value Theorem?

∴ IVT guarantees a value c, $2<c<4$ such that $10<f(c)<20$

(A) $f(x)=13$ has at least one solution in the open interval (2,4)

(B) $f(3)=15$ y = 15 must exist on (2,4) but f(3) does not have to equal 15

(C) f attains a maximum on the open interval (2,4)

(D) $f(x)=10$ at some other value(s) of x other than $x=2$ No

10 is not between 10 and 20.

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2. Let g be a function such that $g(-1)=0$ and $g(2)=5$. Which of the following conditions guarantees that there is an x, $-1<x<2$, for which $g(x)=3$?

$0<(g(x)=3)<5$

$-1<x<2$

(A) g is defined for all x in $(-1,2)$

(B) g is continuous for all x in $[-1,2]$

we need continuity for IVT

(C) g is increasing on $[-1,2]$

(D) there exists an x in $(-1,2)$ such that $g(x)=6$

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y=4 y=4 y=4

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

3. The table above shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times that $f(x)=4$?

IVT applies

(A) One (B) Two (C) Three (D) Four

How many consecutive values in the table "surround" y = 4?

There must be at least three 4s.

There could be much more.

IVT

4. Let f be a function that is continuous on the closed interval $[1,3]$ with $f(1)=10$ and $f(3)=18$. Which of the following statements must be true?

(A) $10 \le f(2) \le 18$ f(2) could be smaller than 10 or larger than 18.

(B) f is increasing on the interval $[1,3]$ Could be true

(C) $f(x)=17$ has at least one solution in the interval $[1,3]$ $10 < 17 < 18$

(D) $f(2)=14$ Could be true

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5. Let f be a function such that $f(1)=-2$ and $f(5)=7$. Which of the following conditions guarantees that $f(c)=0$ for some value c in the open interval $(1,5)$?

$-2 < y=0 < 7$

(A) f is increasing on the closed interval $[1,5]$

IVT needs

(B) f is continuous on the closed interval $[1,5]$

(C) $f(3)=2.5$

(D) f is decreasing on the closed interval $[1,5]$

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t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft / sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft / sec^2)	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

For $0 \le t \le 60$, must there be a time t when $v(t)=-5$? Justify your answer.

- $v(t)$ is continuous
- $v(0)=-20$, $v(60)=10$ $\Big\}$ $-20 < -5 < 10$

$\therefore$ IVT guarantees a value of t on $(0,60)$ such that $v(t)=-5$

x	$f(x)$	$g(x)$
1	6	2
2	9	3
3	10	4
6	–1	6
10	3	11

7. The functions f and g are continuous for all real numbers, and g is strictly increasing. The table above gives values of the functions at selected values of x. The function h is given by $h(x) = g(f(x))$.

Explain why there must be a value r for $1 < r < 3$ such that $h(r) = 8$.

- f, g are continuous $\therefore$ h is continuous
- $\left.\begin{array}{l} h(1) = g(f(1)) = g(6) = 6 \\ h(3) = g(f(3)) = g(10) = 11 \end{array}\right\}$ $6 < 8 < 11$

$\therefore$ IVT guarantees a value of r on $(1,3)$ such that $h(r)=8$.

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t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	-120	-150

8. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a continuous function $v_A(t)$, where t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

Do the data in the table support the conclusion that Train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

- $V_A(t)$ is continuous
- $\left.\begin{array}{l} V_A(5) = 40 \\ V_A(8) = -120 \end{array}\right\}$ $-120 < -100 < 40$

$\therefore$ IVT guarantees a value of t on $(5,8)$ such that $V_A(t) = -100$