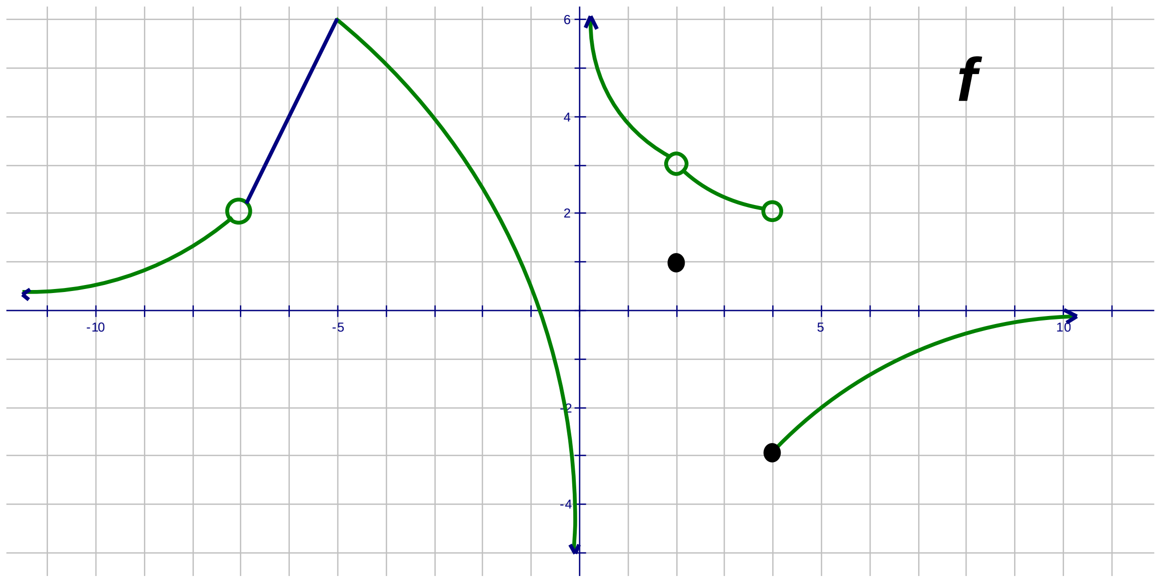
AP Calculus AB
Unit 1 (1.2-1.6) Review - LIMITS

Name: ________Solutions____________

PART I - DO NOT USE A CALCULATOR ON ANY PROBLEM IN THIS SECTION. (Problems 1-37)

Consider the graph of function, f , shown below.



Answer the following questions about function f.

1.) $f(-5) =$ 6

2.) $f(2) =$ 1

3.) $f(4) =$ - 3

4.) $\lim_{x\to -7} f(x) =$ 2

5.) $\lim_{x\to -5} f(x) =$ 6

6.) $\lim_{x\to 2} f(x) =$ 3

7.) $\lim_{x\to 4} f(x) = DNE$

8.) $\lim_{x\to 0} f(x) = DNE$

9.) $\lim_{x\to 0^-} f(x) = -\infty DNE$

10.) $\lim_{x\to 0^+} f(x) = +\infty\ DNE$

11.) $\lim_{x\to 4^+} f(x) = -3$

12.) $\lim_{x\to 4^-} f(x) = 2$

13.) $\lim_{x\to -\infty} f(x) = 0^+$

14.) $\lim_{x\to \infty} f(x) = 0^-$

15.) Use the definition of a continuous function at a number to answer the following.

a. f is not continuous at x = -7 because:__ $f(-7)$ is not defined. ________________________________

b. f is not continuous at x = 2 because:_____ $f(2) \neq \lim_{x\to 2} f(x)$________________________________

c. f is not continuous at x = 4 because:_______ $\lim_{x\to 4} f(x) = DNE$________________________________

DO NOT USE A CALCULATOR

16.) $\lim_{x\to 2}\left(-x^2+4x\right)$

$= \left(-(2)^2+4(2)\right) = 4$

17.) $\lim_{x\to 9^-}\frac{\sqrt{x}-3}{x-9}\begin{cases}\sqrt{9}-3=0\\ 9-9=0\end{cases}$

Do some algebra

$= \lim_{x\to 9^-}\left[\frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}\right]$

$= \lim_{x\to 9^-}\left[\frac{x-9}{(x-9)(\sqrt{x}+3)}\right]$

$= \lim_{x\to 9^-}\left[\frac{1}{\sqrt{x}+3}\right] = \frac{1}{6^-}$

18.) $\lim_{x\to 0}\frac{x}{\tan x}\begin{cases}0\\ \tan 0 = 0\end{cases}$

Do some algebra

$= \lim_{x\to 0}\left[\frac{x}{\sin x}\cdot\frac{\cos x}{1}\right] = (1)(1) = 1$

19.) $\lim_{x\to -2^+}\left(\frac{x}{x+2}\right)$

$= \left(\frac{-2^+}{-2^+ + 2}\right) = \frac{-2^+}{0^+} = -\infty\ DNE$

20.) $\lim_{x\to 0^-}\left(1+\frac{1}{x}\right)$

$= \left(1+\frac{1}{0^-}\right) = -\infty$ DNE

21.) $\lim_{x\to 1}\left(\sin \pi x\right)$

$= \left(\sin \pi(1)\right) = 0$

22.) $\lim_{x\to\infty}\frac{7-6x^5}{x+3}$

$= \lim_{x\to\infty}\frac{-6x^5}{x} = \lim_{x\to\infty}\left(-6x^4\right) = -\infty$

DNE

23.) $\lim_{t\to -\infty}\frac{6-t^3}{7t^3+3}$

$= \lim_{t\to -\infty}\frac{-t^3}{7t^3} = -\frac{1}{7}$

24.) $\lim_{x\to -\infty}\frac{x-2}{x^2+2x+1}$

$= \lim_{x\to -\infty}\frac{x}{x^2+2x} = \lim_{x\to -\infty}\frac{1}{x+2}$

$= \frac{1}{-\infty} = 0^-$

25.) $\lim_{y\to -\infty}\frac{2-y}{\sqrt{7+6y^2}}$

$= \lim_{y\to -\infty}\frac{-y}{\sqrt{6y^2}} = \lim_{y\to -\infty}\frac{-y}{|y|\sqrt{6}}$

$= \frac{1}{\sqrt{6}}$

26.) $\lim_{x\to 2} f(x)$ when

$f(x) = \begin{cases}x^2-3x+6, & x < 2\\ -x^2+3x+2, & x \geq 2\end{cases}$

$\lim_{x\to 2^-} f(x) = 2^2 - 3(2) + 6 = 4$

$\lim_{x\to 2^+} f(x) = -2^2 + 3(2) + 2 = 4$

$\lim_{x\to 2} f(x) = 4$

27.) If a $\neq$ 0, then $\lim_{x\to -a}\frac{x^2-a^2}{x^4-a^4}$ is:

$\lim_{x\to -a}\frac{x^2-a^2}{x^4-a^4}\begin{cases}(-a)^2-a^2=0\\ (-a)^4-a^4=0\end{cases}$

Do some algebra

$\lim_{x\to -a}\frac{x^2-a^2}{(x^2-a^2)(x^2+a^2)}$

$= \lim_{x\to -a}\frac{1}{1(x^2+a^2)} = \frac{1}{2a^2}$

28.) Find a ***c*** such that $f(x)$ is continuous on the entire real line.

$$f(x)=\begin{cases} x^2 & \text{when} \quad x \le 4 \\ \dfrac{c}{x} & \text{when} \quad x > 4 \end{cases}$$

$\lim_{x\to 4^-} f(x) = 4^2 = 16$

$\lim_{x\to 4^+} f(x) = \dfrac{c}{4}$

$\lim_{x\to 4} f(x) = 16$

$c = 64$

29.) Find the x-values (if any) at which f is discontinuous. Label as removable or non-removable.

$$f(x)=\frac{2x+6}{2x^2-18}$$

$$f(x)=\frac{2x+6}{2x^2-18}=\frac{x+3}{x^2-9}$$

$$=\frac{x+3}{(x+3)(x-3)}=\frac{1}{x-3}$$

$f(x)$ is not defined at $x=-3$

$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} f(x) = -\dfrac{1}{6}$

removable discontinuity at $x=-3$ because the limit exists but the function value does not.

30.) Determine all of the vertical asymptotes of f(x):

$$f(x)=\frac{x+2}{x^2-4}$$

$$=\frac{x+2}{(x+2)(x-2)}=\frac{1}{(x-2)}$$

$\lim_{x\to 2^-} \dfrac{1}{(x-2)} = \dfrac{1}{0^-} = -\infty$

$\lim_{x\to 2^+} \dfrac{1}{(x-2)} = \dfrac{1}{0^+} = +\infty$

$x = 2$ is a vertical asymptote

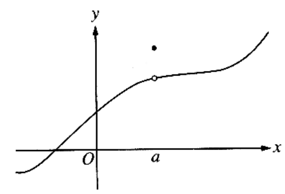
31.) True or False: If f is undefined at x = c, then the limit of f(x) as x approaches c does not exist.

False, the limit could exist and there is a hole in the graph.

32.) True or False: If the $\lim_{x\to c} f(x) = L$ then $f(c) = L$.

False, the function could be discontinuous where $f(c) \neq L$.

33.) The graph of the function f is shown to the right. Which of the following statements is false?



a.) $x = a$ is in the domain of f

b.) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$

c.) $\lim_{x\to a} f(x)$ exists

d.) $\lim_{x\to a} f(x)$ is not equal to $f(a)$

e.) f is continuous at $x = a$ False

34.) $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

$$\frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - (x^2 - 2x + 1)}{\Delta x}$$

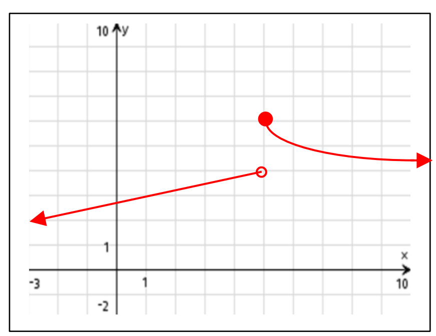
$$=\frac{2x\Delta x + (\Delta x)^2 - 2\Delta x)}{\Delta x} = 2x - 2 + \Delta x$$

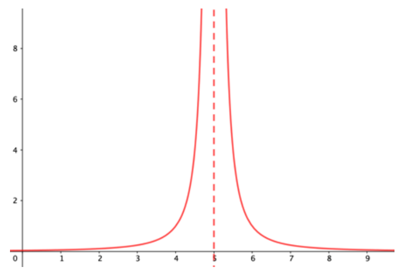
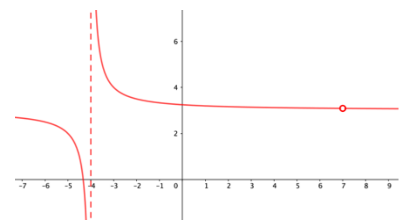
$\lim_{\Delta x\to 0} 2x - 2 + \Delta x = 2x - 2$

35.) On the graph, draw a function that has the following properties:

- A step (or jump) discontinuity at x = 5
- f(5) = 6.

Graphs may vary



36.) Create a function such that the $\lim_{x \to 5}$ does not exist because it is approaching $+\infty$ from both the left and the right. Show both the function and the graph. Answers will vary $f(x) = \dfrac{1}{(x-5)^2}$ 	**37.)** Find a function f(x) such that f(x) has a hole at x = 7 and a vertical asymptote at x = -4. Answers will vary $f(x) = \dfrac{x-7}{(x-7)(x+4)}$ 

Unit 1 (1.2-1.6) Review - LIMITS

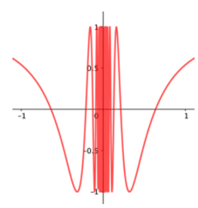
PART II - CALCULATORS MAY BE USED ON THE FIRST PART OF THIS SECTION.

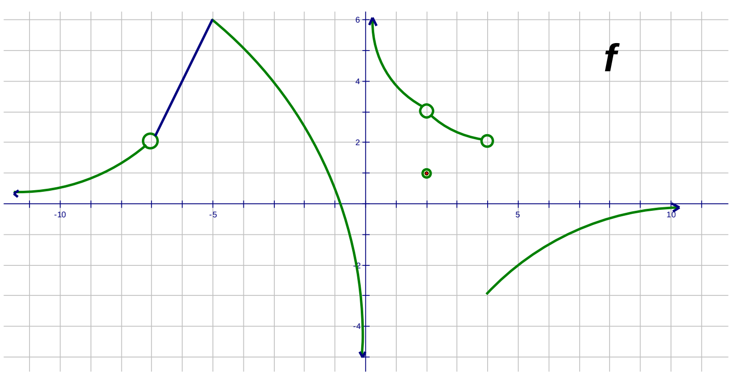
1.) Approximate the limit ***numerically*** by completing the table:

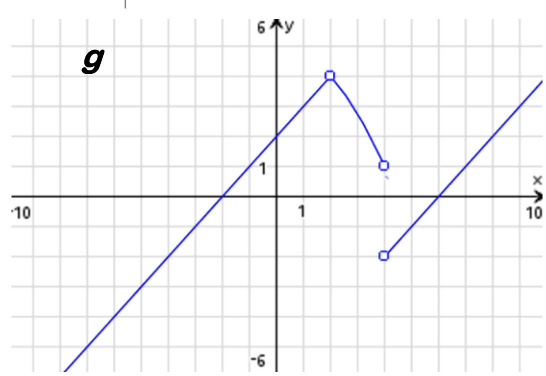
$$\lim_{x \to 2} \frac{x^2}{x-2} = \pm\infty \text{ DNE}$$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	–36.1	–396.01	–3996.001	undefined	4004.001	404.01	44.1

2.) Find the limit: $\lim_{x \to 0} \left(\cos \frac{1}{x}\right) \Rightarrow$ DNE by infinite oscillation







3.) Find $\lim_{x \to 2} f(g(x))$

$u = g(x) \qquad \lim_{x \to 2} g(x) = 4^-$

$\lim_{u \to 4^-} f(u) = 2$

4.) $\lim_{x \to 1} f(x-1) \cdot g(x)$

$\lim_{x \to 1^-} f(x-1) \cdot \lim_{x \to 1^-} g(x)$

$= -\infty \cdot 3 = -\infty$

$\lim_{x \to 1^+} f(x-1) \cdot \lim_{x \to 1^-} g(x)$

$= +\infty \cdot 3 = +\infty$

$\lim_{x \to 1} f(x-1) \cdot \lim_{x \to 1} g(x)$ DNE

5.) $\lim_{x \to 1^+} \dfrac{f(x+1)}{g(x+3)}$

$\lim_{x \to 1^+} f(x+1) = \lim_{(x+1) \to 2^+} f(x+1) = 3$

$\lim_{x \to 1^+} g(x+3) \cdot \lim_{(x+3) \to 4^+} g(x+3) = -2$

$\lim_{x \to 1^+} \dfrac{f(x+1)}{g(x+3)} = \dfrac{3}{-2} = -\dfrac{3}{2}$

6.) No calculator. The piecewise function for $g(x)$ is below. Find the values for *a*, *b*, *c*, and *d* that make $f(x)$ continuous everywhere. Be sure to use the definition of continuity and demonstrate proper notation.

$$f(x)=\begin{cases}\dfrac{x^2+x-2}{x-1}, & x<1\\ a, & x=1\\ b(x-c)^2, & 1<x<4\\ d, & x=4\\ 2x-8, & x>4\end{cases}$$

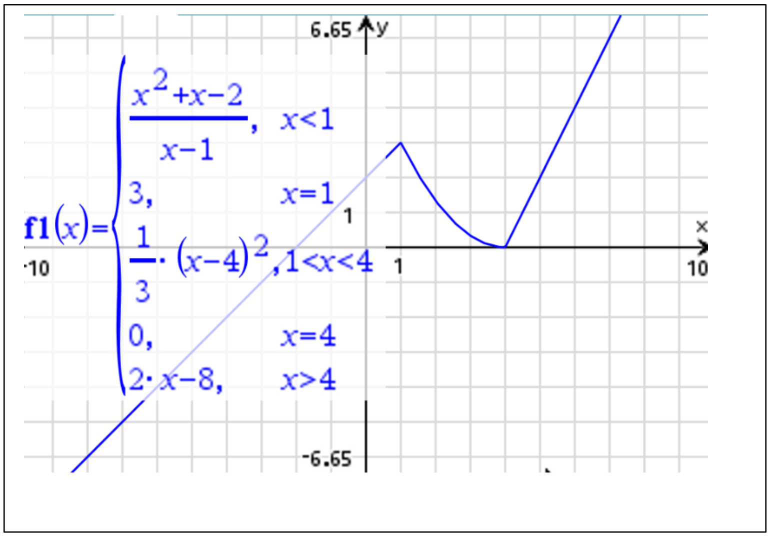
$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-}\left(\frac{x^2+x-2}{x-1}\right)=\lim_{x\to1^-}\frac{(x+2)(x-1)}{x-1}=\lim_{x\to1^-}(x+2)=3$$

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+} b(x-c)^2=b(1-c)^2$$

$$\lim_{x\to4^-} f(x)=\lim_{x\to4^-} b(x-c)^2=b(4-c)^2$$

$$\lim_{x\to4^+} f(x)=\lim_{x\to4^+}(2x-8)=2(4)-8=0$$

$$\begin{cases}3=b(1-c)^2 \rightarrow b=\dfrac{3}{(1-c)^2}\\ 0=b(4-c)^2 \rightarrow b=\dfrac{0}{(4-c)^2}\end{cases}$$



$$\frac{3}{(1-c)^2}=\frac{0}{(4-c)^2}$$

$$3(4-c)^2=0$$

$$\boxed{c=4}$$

Solve for b, $3=b(1-4)^2$

$$3=9b$$

$$\boxed{b=\frac{1}{3}}$$

$$\lim_{x\to1} f(x)=f(1)$$

$$\frac{1}{3}(1-4)^2=a$$

$$\frac{1}{3}(9)=a$$

$$\boxed{a=3}$$

$$\lim_{x\to4} f(x)=f(4)$$

$$\frac{1}{3}(4-4)^2=d$$

$$\boxed{d=0}$$