Skill Builder: Topics 2.1 – The Definition of the Derivative

Find the instantaneous rate of change at the given x-value. Use the form $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$

1. $f(x) = 7\sqrt{x}$ at $x = 2$

$$f'(2) = \lim_{x \to 2} \frac{7\sqrt{x} - 7\sqrt{2}}{x-2}$$

$$= \lim_{x \to 2} \frac{7(\sqrt{x} - \sqrt{2})}{(x-2)} \cdot \frac{(\sqrt{x}+\sqrt{2})}{(\sqrt{x}+\sqrt{2})}$$

$$= \lim_{x \to 2} \frac{7\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}$$

$$= \lim_{x \to 2} \frac{7}{(\sqrt{x}+\sqrt{2})}$$

$$= \frac{7}{\sqrt{2}+\sqrt{2}}$$

$$f'(2) = \frac{7}{2\sqrt{2}}$$

2. $g(x) = 5x - 2x^2$ at $x = -2$

$-10-8=-18$

$$g'(-2) = \lim_{x \to -2} \frac{[5x - 2x^2] - [5(-2) - 2(-2)^2]}{x-(-2)}$$

$$= \lim_{x \to -2} \frac{-2x^2 + 5x + 18}{x+2}$$

$$= \lim_{x \to -2} \frac{-(2x^2 - 5x - 18)}{}$$

$2x^2 - 5x - 18$

$2x^2 - 9x + 4x - 18$

$x(2x-9) + 2(2x-9)$

$(2x-9)(x+2)$

$$= \lim_{x \to -2} \frac{-\cancel{(x+2)}(2x-9)}{\cancel{x+2}}$$

$$= \lim_{x \to -2} (-2x+9)$$

$$= -2(-2)+9$$

$$g'(-2) = 13$$

3. $h(x) = \frac{1}{x+3}$ at $x = 5$

$\frac{1}{8}$

$$\left.\frac{dh}{dx}\right|_{x=5} = \lim_{x\to5} \frac{\left(\frac{1}{x+3} - \frac{1}{5+3}\right)\cdot(x+3)(8)}{(x-5)\cdot(x+3)(8)}$$

$-x+5$

$$= \lim_{x\to5} \frac{8-(x+3)}{8(x-5)(x+3)}$$

$$= \lim_{x\to5} \frac{-\cancel{(x-5)}}{8\cancel{(x-5)}(x+3)}$$

$$\left.\frac{dh}{dx}\right|_{x=5} = \lim_{x\to5} \frac{-1}{8(x+3)}$$

$$\left.\frac{dh}{dx}\right|_{x=5} = \frac{-1}{8(5+3)}$$

$$\left.\frac{dh}{dx}\right|_{x=5} = \frac{-1}{64}$$

4. $k(x) = 10x^2 + 5x - 3$ at $x = -1$

$10-8=2$

$$k'(-1) = \lim_{x\to-1} \frac{[10x^2+5x-3]-[10(-1)^2+5(-1)-3]}{x-(-1)}$$

$$k'(-1) = \lim_{x\to-1} \frac{10x^2+5x-3-2}{x+1}$$

$$= \lim_{x\to-1} \frac{10x^2+5x-5}{x+1}$$

$5(2x^2+1x-1)$

$5[2x^2+2x-x-1]$

$5[2x(x+1)-1(x+1)]$

$5[(x+1)(2x-1)]$

$$= \lim_{x\to-1} \frac{5\cancel{(x+1)}(2x-1)}{\cancel{x+1}}$$

$$= \lim_{x\to-1} 5(2x-1)$$

$$= 5(2(-1)-1)$$

$$k'(-1) = -15$$