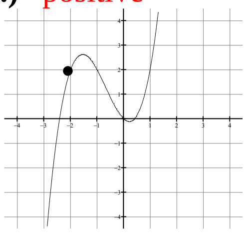
Avon High School **Name** **SOLUTIONS**
AP Calculus AB

Unit 2 Review

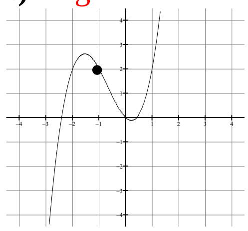
DIFFERENTIATION – Definition of Derivative and Basic Derivative Rules

Determine whether the slope of the tangent line to the curve at the indicated point is positive, negative or zero.

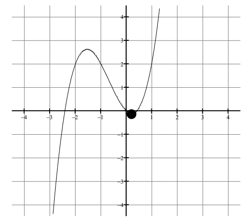
1.) positive



2.) negative



3.) zero



3.) If $f(2)=3$ and $f'(2)=-1$, find the equation of the tangent line when $x=2$.

Note that the slope is -1 and the given point is the ordered pair (2,3). Use the point-slope formula to write the equaiton of your line.

$y-3=-1(x-2)$

If you choose to write this in another form, just be careful to not make any mistakes.

$y=3-(x-2)$

$y=-x+5$

4.) Find the equation of the tangent line to the graph of $f(x)=x^2-2x-3$ when $x=2$.

$f'(x)=2x-2$

$f'(2)=2(2)-2=2$ ← This is your slope

$f(2)=2^2-2(2)-3=-3$ ← This is your point

So, $y-(-3)=2(x-2)$

$y+3=2x-4$

$y=2x-7$

5.) Differentiate $y=\dfrac{3x}{x^2+1}$

You must use the quotient rule for this function.

$$y'=\frac{3(x^2+1)-(3x)(2x)}{(x^2+1)^2}=\frac{3x^2+3-6x^2}{(x^2+1)^2}$$

$$=\frac{-3x^2+3}{(x^2+1)^2}=\frac{-3(x^2-1)}{(x^2+1)^2}=\frac{-3(x-1)(x+1)}{(x^2+1)^2}$$

Note: The simplifed answer(s) on the second line would possible appear in MCQ's only.

6.) For the function $f(t)=\dfrac{t^3+2}{t}$, find the following.

a.) the average rate of change of $f(t)$ on the interval $[1, 4]$

$$f_{aroc}=\frac{f(4)-f(1)}{4-1}=\frac{\frac{(64+2)}{4}-\frac{(1+2)}{1}}{3}=\frac{\frac{33}{2}-3}{3}=\frac{9}{2}$$

b.) the instantaneous rate of change of $f(t)$ when $t=2$

$f(t)=t^2+2t^{-1}$, $f'(t)=2t-\dfrac{2}{t^2}$, $f'(2)=2(2)-\dfrac{2}{2^2}=\dfrac{7}{2}$

7.) Suppose that $h(x)=\dfrac{g(x)}{f(x)}$ and

$g(2)=3,\ g'(2)=-1,\ f(2)=5,\ f'(2)=-2$.

Find $h'(2)$.

$$h'(x)=\frac{g'(x)\cdot f(x)-g(x)\cdot f'(x)}{[f(x)]^2}$$

$$h'(2)=\frac{g'(2)\cdot f(2)-g(2)\cdot f'(2)}{[f(2)]^2}$$

$$h'(2)=\frac{(-1)(5)-(3)(-2)}{5^2}=\frac{-5+6}{25}=\frac{1}{25}$$

8.) Let $f(x)=\begin{cases} ax, & x\le 1 \\ bx^2+x+1, & x>1 \end{cases}$. Find all possible values of a and b such that $f(x)$ is differentiable at $x=1$.

Show proper justification.

In order for a function to be differentiable, it must first be continuous.

$\lim\limits_{x\to 1^-} f(x)=a,\quad \lim\limits_{x\to 1^+} f(x)=b+2$

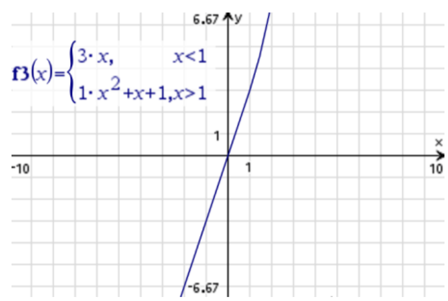
and we know that $a=b+2$

$f'(x)=\begin{cases} a, & x\le 1 \\ 2bx+1, & x>1 \end{cases}$ ← we retain the equals

$\lim\limits_{x\to 1^-} f'(x)=a,\quad \lim\limits_{x\to 1^+} f'(x)=2b+1$

and we also know that $a=2b+1$

$\begin{cases} a=b+2 \\ a=2b+1 \end{cases} \Rightarrow b+2=2b+1 \Rightarrow b=1,\ a=3$



9.) If $f(x)=2x^2+4$, which of the following will calculate the derivative of $f(x)$?

(A) $\dfrac{[2(x+\Delta x)^2+4]-(2x^2+4)}{\Delta x}$

(B) $\lim\limits_{\Delta x\to 0}\dfrac{(2x^2+4+\Delta x)-(2x^2+4)}{\Delta x}$

(C) $\lim\limits_{\Delta x\to 0}\dfrac{[2(x+\Delta x)^2+4]-(2x^2+4)}{\Delta x}$

(D) $\dfrac{(2x^2+4+\Delta x)-(2x^2+4)}{\Delta x}$

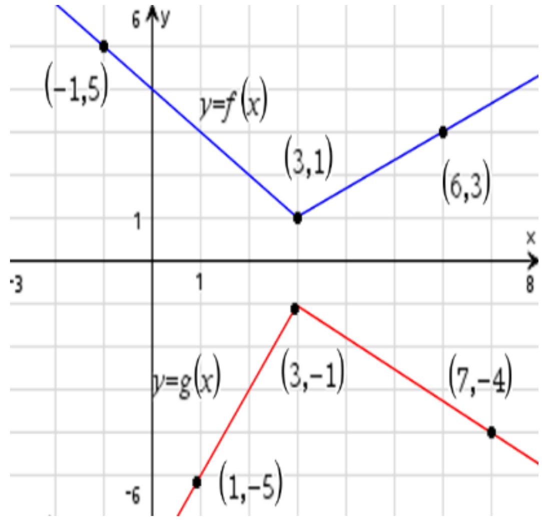
10.) Let $h(x) = f(x) \cdot g(x)$.
Find $h'(2)$.

(A) -2 **(B)** 1

(C) 7 **(D)** -1

$h'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2)$

$= (-1)(-3) + (2)(2) = 3 + 4 = 7$



11.) What is $g'(3)$?

Since the graph of $g(x)$ features a sharp turn at $x = 3$, the derivative does not exist there.

12.) One lazy day, Uncle Si decides to sit under a shade tree next to a road by the swamp and count the number of ducks which cross the road. The data in the table below shows the accumulation of the number of ducks crossing this road at each hour after 9:00am.

Hours after 9AM	0	1	2	3	4	5	6	7
# of ducks that have crossed the road	0	3	8	11	12	21	24	28

Find the following:

a.) Determine the average number of ducks which have crossed the road per hour during Uncle Si's 7-hour observation. Label your result.

$\text{Ducks}_{avg} = \dfrac{28-0}{7-0} = 4 \text{ ducks per hour}$

b.) Estimate the value of $f'(4.5)$ and explain its meaning.

$f'(4.5) \approx \dfrac{21-12}{5-4} = 9$

There are 9 ducks crossing the road per hour at time $t = 4.5$ hours.

13.) $\lim\limits_{\Delta x \to 0} \dfrac{\left[2(-2+\Delta x)^3 - 2(-2+\Delta x) - 2\right] - (-14)}{\Delta x}$ represents $f'(c)$ for a function $f(x)$ and a number c.

Find $f(x)$ and c.

$f(x) = 2x^3 - 2x - 2 \quad c = -2$ Note: $f(-2) = 2(-2)^3 - 2(-2) - 2 = -16 + 4 - 2 = -14$

14.) Suppose that $f(x)$ and $g(x)$ and their derivatives have the following values at $x=-1$ and $x=0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivative of each of the following combinations of $f(x)$ and $g(x)$ at the given value of x.

a.) $3f(x)-g(x), \quad x=-1$

$3f'(-1)-g'(-1)$

$3(2)-(1)$

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b.) $3f(x)\cdot g(x), \quad x=-1$

$3f'(x)\cdot g(x)+3f(x)\cdot g'(x)$

$3f'(-1)\cdot g(-1)+3f(-1)\cdot g'(-1)$

$3(2)(-1)+3(0)(1)$

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c.) $\dfrac{f(x)}{g(x)+2}, \quad x=0$

$$\frac{f'(x)\cdot(g(x)+2)-f(x)(g'(x))}{[g(x)+2]^2}$$

$$\frac{f'(0)\cdot(g(0)+2)-f(0)(g'(0))}{[g(0)+2]^2}$$

$$\frac{(-2)(-3+2)-(-1)(4)}{(-3+2)^2}=\frac{2+4}{1}$$

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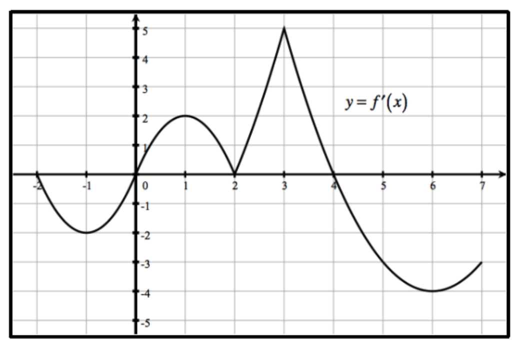
15.) The graph of f', the derivative of f, is shown for $-2<x<7$ and f' is define for all x on the interval $-2<x<7$. What are all values of x for which f has a horizontal tangent line on $-2<x<7$?



(A) $x=0,4$ **(B)** $x=-1,1,6$

(C) $x=0,2,4$ **(D)** $x=2,3$

The graph of $f(x)$ has horizontal tangents lines where the derivative of f is equal to zero. These occur when the graph of $f'(x)$ crosses the x-axis.