Unit 2 Progress Check: FRQ Part A

1

Question 1

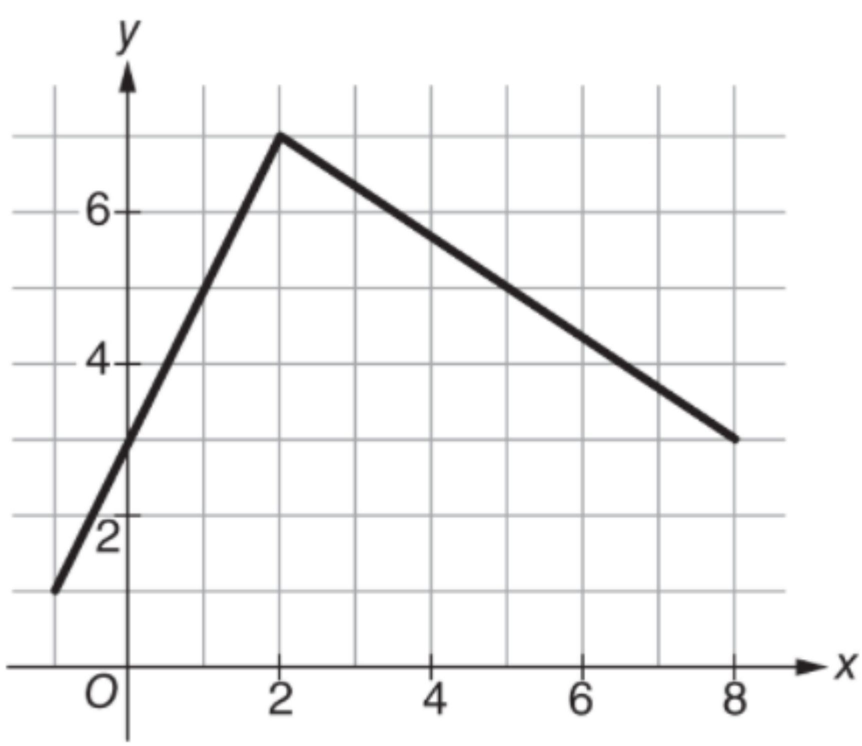
A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

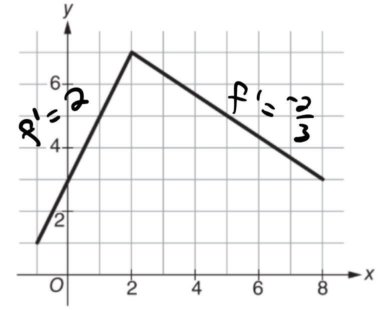
Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f

Let f be the continuous function defined on $[-1, 8]$ whose graph, consisting of two line segments, is shown above. Let g and h be the functions defined by $g(x) = \sqrt{x^2 - x + 3}$ and $h(x) = 5e^x - 9\sin x$.



Graph of f

(a) The function k is defined by $k(x) = f(x)\,g(x)$. Find $k'(0)$.

$K'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$K'(0) = f'(0) \cdot g(0) + f(0) + g'(0)$

$= 2\sqrt{3} + 3(-0.288675)$ +1 +1

$= 2.598$ +1

(b) The function m is defined by $m(x) = \dfrac{f(x)}{2g(x)}$. Find $m'(5)$.

$m'(x) = \dfrac{f'(x) \cdot 2g(x) - f(x) \cdot 2g'(x)}{4g^2(x)}$

$m'(5) = \dfrac{f'(5) \cdot 2g(5) - f(5) \cdot 2g'(5)}{4g^2(5)}$

$= \dfrac{\left(-\frac{2}{3}\right) \cdot 2\sqrt{23} - 5(2 \cdot 0.938315)}{4(\sqrt{23})^2}$ +1 +1

$m'(5) = -0.171$ +1

(c) Find the value of x for $-1 < x < 2$ such that $f'(x) = h'(x)$.

on $-1 < x < 2$, $f'(x) = 2$

$h'(x) = 5e^x - 9\cos(x)$ +1

$2 = 5e^x - 9\cos(x)$

$x \approx 0.622$ +1

For Reference only