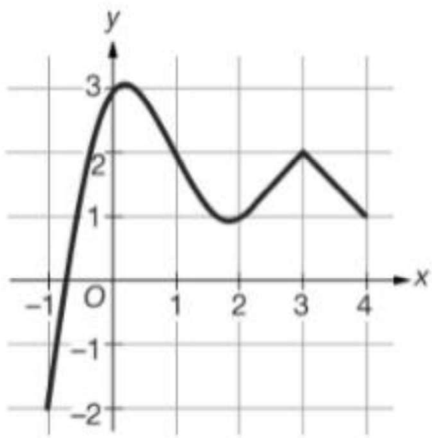
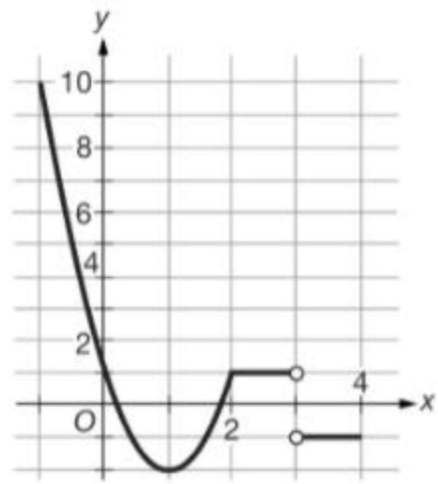
Question 1

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.



Graph of f



Graph of f'

The graphs of the function f and its derivative f' are shown above for $-1 \leq x \leq 4$.

(a) Find the average rate of change of f over the interval $-1 \leq x \leq 4$. For how many values of x in the interval $-1 \leq x \leq 4$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

$$ARC = \frac{f(-1)-f(4)}{-1-4} = \frac{-2-1}{-5} = \frac{-3}{-5} = 3/5$$ +1

Yes the average rate of change is 3/5 and the instantaneous rate of change, f', is equal to 3/5 twice on [0,4] +1

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

$$y-2 = -2(x-1)$$ +1

(c) For each of $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ and $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$, find the value or give a reason why it does not exist.

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = f'(2) = 1$$ +1

$$\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = f'(3) \text{ which does not exist}$$

because $\lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3} \neq \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$

or

because f is not differentiable at $x=3$

+1

(d) Let g be the function defined by $g(x) = e^x f(x)$. Find $g'(0)$.

$g' = e^x \cdot f(x) + e^x \cdot f'(x)$ +1

$g'(0) = e^0 \cdot f(0) + e^0 \cdot f'(0)$

$g'(0) = 1 \cdot 3 + 1 \cdot 1$

$g'(0) = 4$ +1

Question 2

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

t (hours)	0	1	3	6	8
$P(t)$ (passengers)	0	35	204	600	728

Continuous

The number of passengers who have boarded a ship is modeled by the differentiable function P, where t is the number of hours since boarding began. Values of $P(t)$ for selected values of t are given in the table above.

(a) According to the model, what is the average rate at which passengers board the ship, in passengers per hour, over the time interval $1 \le t \le 8$ hours?

$$ARC = \frac{P(1)-P(8)}{1-8} = \frac{35-728}{-7} = \frac{-693}{-7} = 99$$ +1

(b) Write $P'(4.5)$ as the limit of a difference quotient. Use the data in the table to approximate $P'(4.5)$. Show the computations that lead to your answer.

$$P'(4.5) = \lim_{t \to 4.5} \frac{P(t)-P(4.5)}{t-4.5}$$

or

$$P'(4.5) = \lim_{h \to 0} \frac{P(4.5+h)-P(4.5)}{h}$$

+1

$$P'(4.5) \approx \frac{P(3)-P(6)}{3-6} = \frac{204-600}{-3} = \frac{-396}{-3} \approx 132$$ +1

(c) Must there be a time t, for $3 \le t \le 6$, at which $P(t) = 500$? Justify your answer.

I. P is continuous

II. $P(3) = 204$, $P(6) = 600$ } $P(3) < 500 < P(6)$ +1

$\therefore$ The IVT Applies

+1 By the IVT, there must be at least one time t on $3 \le t \le 6$ such that $P(t) = 500$.

(d) The total number of gallons of water used by the passengers on the ship is modeled by the function G defined by $G(t) = 120t\sqrt{t}$ for $0 \le t \le 8$, where t is the number of hours since boarding began. Find $G'(4)$, the rate at which passengers use water, in gallons per hour, at time $t = 4$ hours.

$G(t) = 120t \cdot t^{\frac{1}{2}} = 120t^{3/2}$

$G'(t) = 120 \cdot \frac{3}{2} t^{\frac{1}{2}} = 180\sqrt{t}$ +1

$G'(4) = 180\sqrt{4}$

$= 180 \cdot 2$

$G'(4) = 360$ +1

At time $t = 4$ hours, passengers use water at a rate of 360 gallons per hour.