

Circuit Training - CHAIN Rule

Name Your Mom

Directions: Begin in cell #1. Take the derivative. Search for your answer. Continue in this manner until you complete the circuit. In some cases, you will need to evaluate the derivative, answer a question, or perhaps find the second derivative. Additional paper may be necessary! No technology is needed!

Answer: 2

1 $f(x) = (x^2 + 7)^5, f'(x) = ?$

$$\begin{aligned} f'(x) &= 5(x^2 + 7)^4 \cdot (2x) \\ &= 10x(x^2 + 7)^4 \end{aligned}$$

Answer: $\frac{4}{3}$

19 $f(x) = \tan^2(3x^2), f'(x) = ?$

$$\begin{aligned} f' &= 2[\tan(3x^2)] \cdot \underbrace{\sec^2(3x^2) \cdot 6x}_{\text{chain}} \\ &= 12x \tan(3x^2) \sec^2(3x^2) \end{aligned}$$

Answer: $2x \cos(x^2 + 7)$

4 $f(x) = (x^2 + 7)^{3/2}, f'(x) = ?$

$$\begin{aligned} f'(x) &= \frac{3}{2}(x^2 + 7)^{\frac{1}{2}} \cdot 2x \\ &= 3x\sqrt{x^2 + 7} \end{aligned}$$

Answer: $\frac{-3x \sec^2(3x^2)}{\sqrt{(\tan(3x^2))^3}}$

15 $g(\theta) = \cos(3\theta + \pi), g'\left(\frac{\pi}{4}\right) = ?$

$$\begin{aligned} g' &= -\sin(3\theta + \pi) \cdot 3 \\ g'\left(\frac{\pi}{4}\right) &= -3 \sin\left(\frac{3\pi}{4} + \frac{\pi}{4}\right) = -3 \sin\left(\frac{7\pi}{4}\right) \\ &= -3 \cdot -\frac{\sqrt{2}}{2} \\ &= \frac{3\sqrt{2}}{2} \end{aligned}$$

Answer: $4x \sin(x^2) \cos(x^2)$

6 $g(x) = 2x\sqrt{x^2 + 7}, g'(x) = ?$

$$\begin{aligned} g' &= 2(x^2 + 7)^{\frac{1}{2}} + 2x \cdot \frac{1}{2}(x^2 + 7)^{-\frac{1}{2}} (2x) \\ g' &= 2(x^2 + 7)^{\frac{1}{2}} \left[(x^2 + 7)' + x^2 \right] \\ g' &= \frac{2(2x^2 + 14)}{\sqrt{x^2 + 7}} = \frac{4x^2 + 14}{\sqrt{x^2 + 7}} \end{aligned}$$

Answer: $\frac{8}{9}$

17 $f(x) = \sin(x^2), f''(x) = ?$

$$\begin{aligned} f' &= \cos(x^2) \cdot 2x \\ f'' &= -\sin(x^2) \cdot (2x) \cdot 2x + \cos(x^2) \cdot 2 \\ f'' &= -4x^3 \sin(x^2) + 2\cos(x^2) \end{aligned}$$

Answer: -2

13 $f(x) = \csc\left(\frac{x}{3}\right), f'(\pi) = ?$

$$\begin{aligned} f' &= -\csc\left(\frac{x}{3}\right) \cot\left(\frac{x}{3}\right) \cdot \frac{1}{3} \\ f'(\pi) &= -\frac{1}{3} \csc\left(\frac{\pi}{3}\right) \cot\left(\frac{\pi}{3}\right) \\ &= -\frac{1}{3} \left(\frac{2}{\sqrt{3}}\right) \frac{1}{\sqrt{3}} = -\frac{2}{9} \end{aligned}$$



Answer: $10x(x^2 + 7)^4$

2 $f(x) = 5\sqrt{x^2 + 7}, f'(x) = ?$

$$\begin{aligned} f'(x) &= \frac{5}{2}(x^2 + 7)^{-\frac{1}{2}} (2x) \\ &= \frac{5x}{\sqrt{x^2 + 7}} \end{aligned}$$

Answer: $2x \sec(x^2 + 7) \tan(x^2 + 7)$

9 $f(x) = \sqrt[3]{x^2 + 7} \cos x, f'(x) = ?$

$$\begin{aligned} f' &= \frac{1}{3}(x^2 + 7)^{-\frac{2}{3}} (2x) \cos x + (x^2 + 7)^{\frac{1}{3}} \cdot (-\sin x) \\ &= (x^2 + 7)^{-\frac{2}{3}} \left[\frac{2x}{3} \cos x - (x^2 + 7) \sin x \right] \\ &= \frac{\frac{2}{3}x \cos x - x^2 \sin x - 7 \sin x}{\sqrt[3]{x^2 + 7}} \end{aligned}$$



Answer: $\frac{5x}{\sqrt{x^2 + 7}}$

3 $g(x) = \sin(x^2 + 7), g'(x) = ?$

$$g'(x) = \cos(x^2 + 7) \cdot 2x$$

Answer: $3x\sqrt{x^2 + 7}$

5 $y = \sin^2(x^2)$, $y' = ?$
Double chain

$$y' = 2[\sin(x^2)]^1 \cdot (\cos(x^2)) \cdot 2x$$

$$= 4x \sin(x^2) \cos(x^2)$$

Answer: $2 \cos(x^2) - 4x^2 \sin(x^2)$

18 For what value of t does $y = \frac{\sqrt{3t-2}}{t}$ have a horizontal tangent?

$$y' = \frac{2\sqrt{3t-2} \cdot t - (3t-2) \cdot \sqrt{3t-2}}{t^2 \cdot 2\sqrt{3t-2}}$$

$$y' = \frac{3t-2(3t-2)}{2t^2 \sqrt{3t-2}} = \frac{3t-6t+4}{2t^2 \sqrt{3t-2}} = \frac{-3t+4}{2t^2 \sqrt{3t-2}}$$

$$\begin{cases} y' = 0 \\ -3t+4 = 0 \\ t = \frac{4}{3} \end{cases}$$

Answer: $\frac{2x \cos x - 7 \sin x - x^2 \sin x}{3\sqrt{(x^2+7)^2}}$

10 The functions $f(x)$ and $g(x)$ are differentiable with select values in the table. Let $h(x) = f(g(x))$. What is $h'(2)$?

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	-4/3	5	0
2	1/2	5	-1	3
5	0	1.2	π	10

$$h' = f'(g(x)) \cdot g'(x)$$

$$h'(2) = f'(g(2)) \cdot g'(2) = f'(-1) \cdot 3$$

$$= -\frac{4}{3} \cdot 3 = -4$$

Answer: $\frac{3\sqrt{2}}{2}$

16 $r(t) = \frac{3}{(t^2-2t)^2} = 3(t^2-2t)^{-2}$, $r'(-1) = ?$

$$r'(t) = -6(t^2-2t)^{-3} \cdot (2t-2)$$

$$r'(-1) = \frac{-6(-1-2)}{((-1)^2-2(-1))^3} = \frac{-6(-1)}{(3)^3} = \frac{24}{27} = \frac{8}{9}$$

Answer: 21

12 Given: $g(x) = 3x^2 - 6x$, $h(x) = \sqrt{x}$
If $f(x) = h(g(x))$, then $f'(-1) = ?$

$$g(-1) = 3(-1)^2 - 6(-1) \quad g' = 6x-6$$

$$= 3 + 6 \quad h' = \frac{1}{2\sqrt{x}}$$

$$g(-1) = 9 \quad f' = h'(g(x)) \cdot g'(x)$$

$$f'(-1) = h'(g(-1)) \cdot g'(-1)$$

$$= h'(9) \cdot (-12) \quad = \frac{1}{2}(-12) = -6$$

Answer: -4

11 $g(t) = (t^3 - 2)^7$, $g'(1) = ?$

$$g'(t) = 7(t^3-2)^6 \cdot 3t^2$$

$$g'(1) = 7(1^3-2)^6 \cdot 3(1)^2$$

$$= 7(1) \cdot 3$$

$$= 21$$

Answer: $-\csc x(\cot^2 x + \csc^2 x)$

8 $h(x) = \sec(x^2 + 7)$, $h'(x) = ?$

$$h' = \sec(x^2+7) \tan(x^2+7) \cdot 2x$$

Answer: $12x \tan(3x^2) \sec^2(3x^2)$

20 The functions $f(x)$ and $g(x)$ are differentiable with select values in the table. Let $p(x) = f(x) \cdot g(f(x))$. What is $p'(5)$?

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	1/3	π	-4	6
5	2	1/4	0	-6

$$p'(5) = f'(5) \cdot g(f(5)) + f(5) \cdot g'(f(5)) \cdot f'(5)$$

$$= \left(\frac{1}{4}\right) \cdot g(2) + (2) g'(2) \cdot \left(\frac{1}{4}\right)$$

$$= \frac{1}{4} \cdot (-4) + 2 \cdot 4 \cdot \left(\frac{1}{4}\right)$$

$$= -1 + 2 = 1$$

Answer: $-\frac{2}{9}$

14 $g(x) = \frac{1}{\sqrt{\tan(3x^2)}} = (\tan(3x^2))^{-\frac{1}{2}}$, $g'(x) = ?$

$$g = \frac{1}{2}(\tan(3x^2))^{-\frac{3}{2}} \cdot \sec^2(3x^2) \cdot 6x$$

$$g' = \frac{-3 \cdot \sec^2(3x^2)}{\sqrt{\tan^3(3x^2)}}$$

Answer: $\frac{4x^2+14}{\sqrt{x^2+7}}$

7 $y = \frac{\cos x}{\sin^2 x}$, $\frac{dy}{dx} = ?$

$$y' = -\csc^2 x \cdot \csc x + \cot x (-\cot x \csc x)$$

$$y' = -\csc^3 x [\csc^2 x + \cot^2 x]$$

GCF