# Unit 3 Progress Check FRQ Part A

FRQ 1

| $x$ | $-3$ | $0$ | $3$ | $6$ |
|---|---|---|---|---|
| $f(x)$ | $-5$ | $4$ | $1$ | $7$ |
| $f'(x)$ | $-1$ | $2$ | $-2$ | $4$ |

The table above gives values of a twice-differentiable function $f$ and its first derivative $f'$ for selected values of $x$. Let $g$ be the function defined by $g(x) = f(x^2 - x)$.

(a) What is the value of $g'(3)$?

+1 $g'(x) = \underbrace{f'(x^2-x)}_{\text{LINK}} \cdot \underbrace{(2x-1)}_{\text{LINK}}$ (chain)

$g'(3) = f'(3^2-3)\cdot(2\cdot3-1)$

$= f'(6)(5)$

$= 4\cdot5$

+1 $g'(3) = 20$

(b) It is known that $g''(0) = -1$. What is the value of $f''(0)$?

+2 $g''(x) = \underbrace{\underbrace{f''(x^2-x)(2x-1)}_{\text{LINK}}\cdot\underbrace{(2x-1)}_{\text{LINK}} + f'(x^2-x)\cdot 2}_{\text{product}}$

$g''(0) = f''(0^2-0)(2\cdot0-1)(2\cdot0-1) + f'(0^2-0)\cdot2$

$-1 = f''(0)(-1)(-1) + f'(0)\cdot2$

$-1 = f''(0) + 2\cdot2$

$-5 = f''(0)$ +1

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(c) Is there a value $c$, for $0 < c < 3$, such that $g(c) = 5$ ? Justify your answer.

IVT

$f$ is differentiable

$\therefore$ $f$ is continuous

$\therefore$ $g$ is the composition of two continuous functions

$\therefore$ $g$ is continuous +1

$g(0) < 5 < g(3)$

$g(0) = f(0^2 - 0) = f(0) = 4$

$g(3) = f(3^2 - 3) = f(6) = 7$

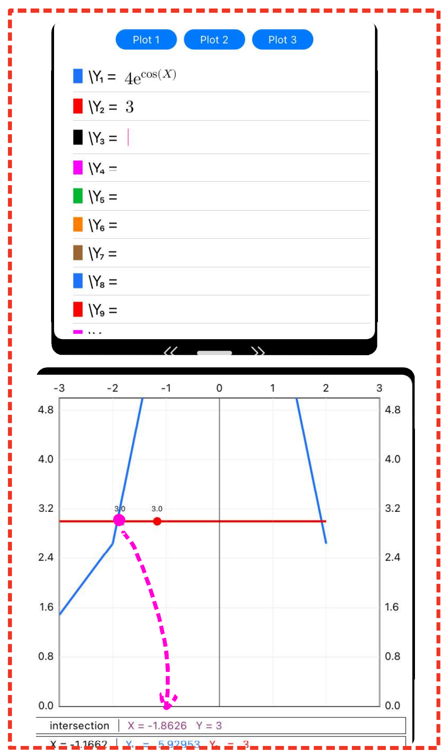
$g(0) < 5 < g(3)$

+1

The IVT guarantees a value of $c$ on $0 < c < 3$ such that $g(c) = 5$

(d) Let $h$ be the function with derivative given by $h'(x) = 4e^{\cos x}$. At what value of $x$ in the interval $-3 \le x \le 0$ does the instantaneous rate of change of $h$ equal the average rate of change of $f$ over the interval $-3 \le x \le 0$ ?

Reference only




$$\text{ARC of } f = \frac{f(-3) - f(0)}{-3 - 0} = \frac{-5 - 4}{-3} = \frac{-9}{-3} = 3$$ +1

$h'(x) = 3$ on $[-3, 0]$ at $x = -1.863$

The instantaneous rate of change of $h$ equals the average rate of change of $f$ at $x = -1.863$ on $[-3, 0]$

+1