

Unit 3 Progress Check FRQ Part B

FRQ 1

Consider the curve given by the equation $(2y + 1)^3 - 24x = -3$.

(a) Show that $\frac{dy}{dx} = \frac{4}{(2y+1)^2}$.

$$\frac{d}{dx} \left((2y+1)^3 - 24x = -3 \right)$$

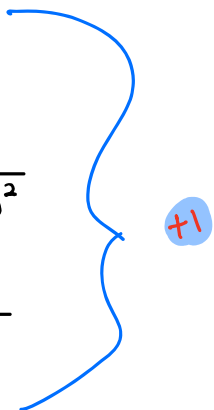
$$3(2y+1)^2 \left(2 \frac{dy}{dx} \right) - 24 = 0 \quad +1$$

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$$3(2y+1)^2 \left(2 \frac{dy}{dx} \right) = 24$$

$$\frac{dy}{dx} = \frac{24 \cdot 4}{3 \cdot 2 (2y+1)^2}$$

$$\frac{dy}{dx} = \frac{4}{(2y+1)^2}$$



(b) Write an equation for the line tangent to the curve at the point $(-1, -2)$.

Point $(-1, -2)$ slope = $\frac{4}{9}$

Tangent line

$$\left. \frac{dy}{dx} \right|_{(-1, -2)} = \frac{4}{(2(-2)+1)^2}$$

$$y + 2 = \frac{4}{9}(x + 1)$$

$$= \frac{4}{(-3)^2}$$

$$\left. \frac{dy}{dx} \right|_{(-1, -2)} = \frac{4}{9} \quad +1$$

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FRQ 1

Consider the curve given by the equation $(2y + 1)^3 - 24x = -3$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

$$\frac{d}{dx} \left(\frac{dy}{dx} = \frac{4}{(2y+1)^2} \right)$$

$$\frac{d}{dx} \left(\frac{dy}{dx} = 4(2y+1)^{-2} \right)$$

+2

$$\frac{d^2y}{dx^2} = 4(-2)(2y+1)^{-3} \left(2 \frac{dy}{dx} \right)$$

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Best answer.

$$\left. \frac{d^2y}{dx^2} \right|_{(-1,-2)} = -8(2(-2)+1)^{-3} \cdot 2 \cdot \frac{4}{9} \quad (\text{from part b})$$

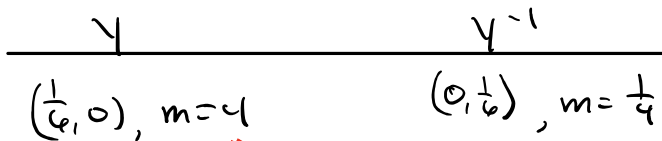
+1

$$= -8(-3)^{-3} \cdot \frac{8}{9}$$

$$= \frac{-64}{-27 \cdot 9}$$

$$\frac{d^2y}{dx^2} = \frac{64}{243}$$

(d) The point $(\frac{1}{6}, 0)$ is on the curve. Find the value of $(y^{-1})'(0)$.



$$(y^{-1})'(0) = \frac{1}{4}$$

+1

$$\left. \frac{dy}{dx} \right|_{\substack{x=\frac{1}{6} \\ y=0}} = \frac{4}{(2 \cdot 0 + 1)^2} = \frac{4}{1^2} = 4$$

+1

Unit 3 Progress Check FRQ Part B

FRQ 2

x	-3	-2	-1	1
$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

Continuous

The table above gives values of the differentiable function f and its derivative for selected values of x .

(a) Let g be the function defined by $g(x) = \frac{f(x^2)}{e^x}$. Find $g'(-1)$.

$$g'(x) = \frac{f'(x^2) \cdot 2x \cdot e^x - f(x^2) \cdot e^x}{(e^x)^2}$$

Quotient Rule +1

$$g'(-1) = \frac{f'(1) \cdot 2(-1) \cdot e^{-1} - f(1) \cdot e^{-1}}{(e^{-1})^2}$$

$$g'(-1) = \frac{\frac{4}{3} \cdot (-2) \cdot e^{-1} - \frac{2}{3} e^{-1}}{e^{-2}}$$

Best answer +1

$$= \frac{e^{-1} \left(-\frac{8}{3} - \frac{2}{3} \right)}{e^{-2}}$$

$$= e^{-\frac{10}{3}}$$

$$= -\frac{10e}{3}$$

(b) Let h be the function defined by $h(x) = f(f(-2x))$. Find $h'(1)$.

$$h'(x) = f'(f(-2x)) \cdot f'(-2x) \cdot (-2)$$

+2

$$h'(1) = f'(f(-2)) \cdot f'(-2) \cdot (-2)$$

$$= f'(-3) \cdot \frac{1}{3} \cdot (-2)$$

$$= -1 \cdot \frac{1}{3} \cdot (-2)$$

$$h'(1) = \frac{2}{3}$$

+1

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FRQ 2

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$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

The table above gives values of the differentiable function f and its derivative for selected values of x .

(c) Let k be the function defined by $k(x) = f(x) \cdot \arcsin(\frac{x}{2})$. Find $k'(-1)$.

Product Rule +1

$$k'(x) = f'(x) \cdot \arcsin(\frac{x}{2}) + f(x) \cdot \frac{1}{\sqrt{1 - (\frac{x}{2})^2}} \left(\frac{1}{2}\right)$$

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$$k'(-1) = f'(-1) \arcsin\left(\frac{-1}{2}\right) + f(-1) \cdot \frac{1}{\sqrt{1 - (\frac{-1}{2})^2}} \cdot \frac{1}{2}$$

$$k'(-1) = \frac{6}{5} \arcsin\left(\frac{-1}{2}\right) + (-2) \cdot \frac{1}{\sqrt{1 - (\frac{-1}{2})^2}} \cdot \frac{1}{2}$$

Best answer

$$= \frac{6}{5} \left(-\frac{\pi}{6}\right) + \left(-2 \cdot \frac{1}{2}\right) \frac{1}{\sqrt{1 - \frac{1}{4}}}$$

$$= \frac{-\pi}{5} - \frac{1}{\sqrt{\frac{3}{4}}}$$

$$= \frac{-\pi}{5} - \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{-\pi}{5} - \frac{2}{\sqrt{3}}$$

