Unit 3 Progress Check FRQ Part B

FRQ 1

Consider the curve given by the equation $(2y+1)^3 - 24x = -3$.

(a) Show that $\frac{dy}{dx} = \frac{4}{(2y+1)^2}$.

$$\frac{d}{dx}\left((2y+1)^3 - 24x = -3\right)$$

$$3(2y+1)^2\left(2\frac{dy}{dx}\right) - 24 = 0 \quad +1$$

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$$3(2y+1)^2\left(2\frac{dy}{dx}\right) = 24$$

$$\frac{dy}{dx} = \frac{\cancel{24}\ 4}{\cancel{3\cdot 2}\,(2y+1)^2}$$

$$\frac{dy}{dx} = \frac{4}{(2y+1)^2}$$

+1

(b) Write an equation for the line tangent to the curve at the point $(-1, -2)$.

Point $(-1,-2)$ Slope $= \frac{4}{9}$

Tangent line

$$\left.\frac{dy}{dx}\right|_{(-1,-2)} = \frac{4}{(2(-2)+1)^2}$$

$$= \frac{4}{(-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(-1,-2)} = \frac{4}{9} \quad +1$$

$$y + 2 = \frac{4}{9}(x+1) \quad +1$$

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FRQ 1

Consider the curve given by the equation $(2y+1)^3 - 24x = -3$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(-1,-2)$.

$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{4}{(2y+1)^2}\right)$$

$$\frac{d}{dx}\left(\frac{dy}{dx} = 4(2y+1)^{-2}\right)$$

+2 $$\frac{d^2y}{dx^2} = 4(-2)(2y+1)^{-3}\left(2\frac{dy}{dx}\right)$$

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Best answer.

$$\boxed{\left.\frac{d^2y}{dx^2}\right|_{(-1,-2)} = -8(2(-2)+1)^{-3}\cdot 2\cdot\frac{4}{9} \quad +1}$$

(from part b)

$$= -8(-3)^{-3}\cdot\frac{8}{9}$$

$$= \frac{-64}{-27\cdot 9}$$

$$\frac{d^2y}{dx^2} = \frac{64}{243}$$

(d) The point $\left(\frac{1}{6}, 0\right)$ is on the curve. Find the value of $\left(y^{-1}\right)'(0)$.

y	y^{-1}
$\left(\frac{1}{6}, 0\right)$, $m=4$	$\left(0, \frac{1}{6}\right)$, $m=\frac{1}{4}$

$$\left(y^{-1}\right)'(0) = \frac{1}{4}$$

+1

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{6},\ y=0} = \frac{4}{(2\cdot 0+1)^2} = \frac{4}{1^2} = 4$$

+1

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FRQ 2

x	-3	-2	-1	1
$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

Continuous

The table above gives values of the differentiable function f and its derivative for selected values of x.

(a) Let g be the function defined by $g(x) = \frac{f(x^2)}{e^x}$. Find $g'(-1)$.

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$$g'(x) = \frac{f'(x^2)\cdot 2x \cdot e^x - f(x^2)\cdot e^x}{(e^x)^2}$$

Quotient Rule +1

$$g'(-1) = \frac{f'(1)\cdot 2(-1)\cdot e^{-1} - f(1)\cdot e^{-1}}{[e^{-1}]^2}$$

$$\boxed{g'(-1) = \frac{\frac{4}{3}\cdot(-2)\cdot e^{-1} - \frac{2}{3}e^{-1}}{e^{-2}}}$$ +1

Best answer

$$= \frac{e^{-1}\left(-\frac{8}{3} - \frac{2}{3}\right)}{e^{-2}}$$

$$= e\cdot -\frac{10}{3}$$

$$= \frac{-10e}{3}$$

(b) Let h be the function defined by $h(x) = f(f(-2x))$. Find $h'(1)$.

$$h'(x) = f'(f(-2x))\cdot f'(-2x)\cdot(-2)$$ +2

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$$h'(1) = f'(f(-2))\cdot f'(-2)\cdot(-2)$$

$$= f'(-3)\cdot \frac{1}{3}(-2)$$

$$= -1\cdot\frac{1}{3}\cdot(-2)$$

$$h'(1) = \frac{2}{3}$$ +1

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FRQ 2

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The table above gives values of the differentiable function f and its derivative for selected values of x.

(c) Let k be the function defined by $k(x) = f(x) \cdot \arcsin\left(\frac{x}{2}\right)$. Find $k'(-1)$.

Product Rule +1

$$K'(x) = f'(x) \cdot \arcsin\left(\tfrac{1}{2}x\right) + f(x) \cdot \frac{1}{\sqrt{1-\left(\frac{1}{2}x\right)^2}}\left(\tfrac{1}{2}\right)$$

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$$K'(-1) = f'(-1)\arcsin\left(\tfrac{-1}{2}\right) + f(-1) \cdot \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\,\frac{1}{2}$$

$$\boxed{K'(-1) = \frac{6}{5}\arcsin\left(\tfrac{-1}{2}\right) + (-2) \cdot \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\,\frac{1}{2}}$$

+1

Best answer

$$= \frac{6}{5}\left(\frac{-\pi}{6}\right) + \left(-2 \cdot \frac{1}{2}\right)\frac{1}{\sqrt{1-\frac{1}{4}}}$$

$$= \frac{-\pi}{5} - \frac{1}{\sqrt{\frac{3}{4}}}$$

$$= \frac{-\pi}{5} - \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{-\pi}{5} - \frac{2}{\sqrt{3}}$$

$\arcsin\left(\frac{-1}{2}\right) = \frac{-\pi}{6}$