Unit 3 Progress Check MC Online

Scratch paper

Question 1

If $g(x) = \ln x$ and f is a differentiable function of x, which of the following is equivalent to the derivative of $f(g(x))$ with respect to x ?

Continuous

$f'(g(x)) \cdot g'(x)$

$= f'(\ln x) \cdot \frac{1}{x}$

(A) $f'\left(\frac{1}{x}\right)$

(B) $\frac{f'(x)}{x}$

(C) $f'(\ln x)$

(D) $\frac{f'(\ln x)}{x}$

Question 2

For which of the following functions is the chain rule an appropriate method to find the derivative with respect to x ?

I. $y = \sin(3x^2)$ → $y = \sin(3x^2)$ (INSIDE / outside)

II. $y = e^x \tan x$ → $e^x \tan x = y$ (Product)

III. $y = \frac{1}{8x^4 - 2x}$ → $y = \frac{1}{8x^4-2x}$ (Quotient) or $y = (8x^4-2x)^{-1}$ (Chain; INSIDE / outside)

(A) I only

(B) II only

(C) III only

(D) I and III only

Question 3

Let f be a differentiable function. If $h(x) = (1 + f(3x))^2$, which of the following gives a correct process for finding $h'(x)$?

$h'(x) = 2(1 + f(3x))^1 \cdot f'(3x) \cdot 3$ (Link / Link / Link)

(A) $h'(x) = 2(1 + f(3x))$

(B) $h'(x) = 2(1 + f(3x)) \cdot f'(3x)$

(C) $h'(x) = 2(1 + f(3x)) \cdot f'(x)$

(D) $h'(x) = 2(1 + f(3x)) \cdot f'(3x) \cdot 3$

Unit 3 Progress Check MC Online

Scratch paper

Question 4

What is the slope of the line tangent to the curve $y^3 - xy^2 + x^3 = 5$ at the point $(1, 2)$?

FIND $\left.\frac{dy}{dx}\right|_{(1,2)}$

(A) $\frac{1}{10}$

(B) $\frac{1}{8}$

(C) $\frac{5}{12}$

(D) $\frac{11}{4}$

$3y^2\frac{dy}{dx} - \left[1\cdot y^2 + x\cdot 2y\frac{dy}{dx}\right] + 3x^2 = 0$

@ $(1,2)$ $3(2)^2\frac{dy}{dx} - \left[(2)^2 + 1\cdot 2\cdot 2\cdot\frac{dy}{dx}\right] + 3(1)^2 = 0$

$12\frac{dy}{dx} - 4 - 4\frac{dy}{dx} + 3 = 0$

$8\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{8}$

or

$3y^2\frac{dy}{dx} - \left[1\cdot y^2 + x\cdot 2y\frac{dy}{dx}\right] + 3x^2 = 0$

$\frac{dy}{dx}\left[3y^2 - 2xy\right] = y^2 - 3x^2$

$\frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}$

@ $(1,2)$ $\frac{dy}{dx} = \frac{(2)^2 - 3(1)^2}{3(2)^2 - 2(1)(2)}$

$\frac{dy}{dx} = \frac{4-3}{12-4} = \frac{1}{8}$

Question 5

If $\sin(x + y) = 3x - 2y$, then $\frac{dy}{dx} =$

(A) $\frac{3-\cos(x+y)}{2}$

(B) $\frac{1-\cos(x+y)}{\cos(x+y)}$

(C) $\frac{3}{2+\cos(x+y)}$

(D) $\frac{3-\cos(x+y)}{2+\cos(x+y)}$

$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 3 - 2\frac{dy}{dx}$

LINK LINK

$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = 3 - 2\frac{dy}{dx}$

$\frac{dy}{dx}\left(2 + \cos(x+y)\right) = 3 - \cos(x+y)$

$\frac{dy}{dx} = \frac{3 - \cos(x+y)}{2 + \cos(x+y)}$

Question 6

$f(-2) = 3$	$f'(-2) = 4$	$g(4) = 5$	$g'(4) = 2$

The point $(-2, 4)$ lies on the curve in the xy-plane given by the equation $f(x)g(y) = 17 - x - y$ where f is a differentiable function of x and g is a differentiable function of y. Selected values of f, f', g, and g' are given in the table above. What is the value of $\frac{dy}{dx}$ at the point $(-2, 4)$?

(A) -27

(B) $-\frac{11}{3}$

(C) -3

(D) $-\frac{4}{7}$

$\frac{d}{dx}\left(f(x)\cdot g(y) = 17 - x - y\right)$

$f'(x)\cdot g(y) + f(x)\cdot g'(y)\cdot\frac{dy}{dx} = -1 - \frac{dy}{dx}$

@ $(-2,4)$ $f'(-2)\cdot g(4) + f(-2)\cdot g'(4)\frac{dy}{dx} = -1 - \frac{dy}{dx}$

$4\cdot 5 + 3\cdot 2\frac{dy}{dx} = -1 - \frac{dy}{dx}$

$21 + 6\frac{dy}{dx} = -\frac{dy}{dx}$

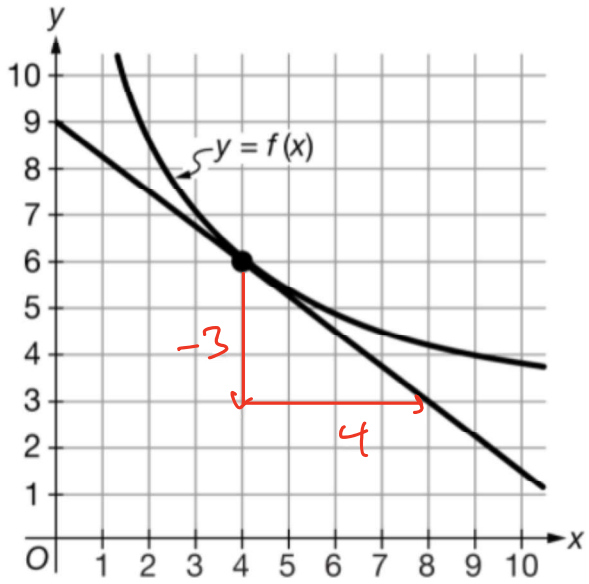
$21 = -7\frac{dy}{dx}$

$-3 = \frac{dy}{dx}$

Unit 3 Progress Check MC Online

Scratch paper

Question 7



The graph of the decreasing differentiable function f is shown above. Also shown is the line tangent to the graph of f at the point $(4, 6)$. Let g be the inverse of f. Which of the following statements about g' is true?

f | g
$(4,6), m = -\frac{3}{4}$ | $(6,4), m = -\frac{4}{3}$

(A) $g'(4) = -\frac{4}{3}$

(B) $g'(4) = -\frac{3}{4}$

(C) $g'(6) = -\frac{4}{3}$

(D) $g'(6) = -\frac{3}{4}$

Question 8

1-1 ∴ has inverse

Let f be the increasing function defined by $f(x) = x^3 + 2x^2 + 4x + 5$, where $f(-1) = 2$. If g is the inverse function of f, which of the following is a correct expression for $g'(2)$?

f | g
$(-1,2), m = 3$ | $(2,-1), m = \frac{1}{3}$

(A) $g'(2) = \frac{1}{f'(2)}$

(B) $g'(2) = \frac{1}{f'(-1)}$

(C) $g'(2) = f'(-1)$

(D) $g'(2) = f'(2)$

$f' = 3x^2 + 4x + 4$

$f'(-1) = 3(-1)^2 + 4(-1) + 4$

$f'(-1) = 3$

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Scratch paper

Question 9

x	0	2	4
$f(x)$	8	5	2
$f'(x)$	-1	-2	-5

Continuous 1-1 ∴ inverse

The table above gives selected values for a differentiable and decreasing function f and its derivative. If $g(x) = f^{-1}(x)$ for all x, which of the following is a correct expression for $g'(2)$?

f g
(4, 2), m = -5 (2, 4), m = $-\frac{1}{5}$

(A) $g'(2) = f'(2) = -2$

(B) $g'(2) = \frac{1}{f'(2)} = -\frac{1}{2}$

(C) $g'(2) = \frac{1}{f'(4)} = -\frac{1}{5}$

(D) $g'(2) = -\frac{f'(2)}{(f(2))^2} = \frac{2}{25}$

Question 10

$\frac{d}{dx}(\sin^{-1}x)|_{x=\frac{1}{2}} =$ $\frac{1}{\sqrt{1-x^2}}\Big|_{x=\frac{1}{2}} = \frac{1}{\sqrt{1-(\frac{1}{2})^2}}$

(A) $\frac{1}{1+\left(\frac{1}{2}\right)^2}$

(B) $\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$

(C) $\cos^{-1}\left(\frac{1}{2}\right)$

(D) $-\csc\left(\frac{1}{2}\right)\cot\left(\frac{1}{2}\right)$

Question 11

$\frac{d}{dx}(\cos^{-1}x) =$

(A) $-\frac{1}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $-\sin^{-1}x$

(D) $-\cos^{-2}x$

Unit 3 Progress Check MC Online

Scratch paper

Question 12

Which of the following methods can be used to find the derivative of $y = \arcsin x$ with respect to x ?

I. Use the quotient rule to differentiate $\frac{1}{\sin x}$. $\neq \arcsin x$

II. Use the chain rule to differentiate $\sin(\arcsin x) = x$. True

III. Use implicit differentiation to differentiate the function y in the relation $\sin y = x$ with respect to x. True

(A) I only

(B) III only

(C) II and III only

(D) I, II, and III

Question 13

Which of the following expressions can be differentiated using the product rule?

(A) $\cos(\sqrt{x})$ Chain (inside / outside)

(B) $x^2 \tan^{-1} x$ product

(C) $x^4 + \arcsin x$ Sum

(D) $(8x^3 - 5x + 2)^{\pi}$ Chain (INSIDE / outside)

Question 14

Which of the following requires the use of implicit differentiation to find $\frac{dy}{dx}$?

(A) $y - x^2 - 3x + 5 = 0 \Rightarrow y = x^2 + 3x - 5$ Explicit

(B) $y = \ln(3 + x) + x^2$ Explicit

(C) $y = \ln(y + x) + x^2$

(D) $y = \frac{x^3 - 4}{3x + 2}$ Explicit

Unit 3 Progress Check MC Online

Scratch paper

Question 15

For which of the following functions would the quotient rule be considered the best method for finding the derivative?

Ⓐ $y = (2x+1)^{-\frac{1}{2}}$ Chain

inside, outside

Ⓑ $y = \frac{2x+1}{x} = \frac{2x}{x} + \frac{1}{x} = 2 + x^{-1}$ Power Rule

Ⓒ $y = \sin^{-1}(2x+1)$ Chain

Ⓓ $y = \frac{\sin(2x+1)}{2x+1}$ outside, inside } Quotient

Question 16

If $y = 2\ln x$, then $\frac{d^4y}{dx^4} =$

Ⓐ $\frac{2}{x}$ $\frac{dy}{dx} = 2 \cdot \frac{1}{x} = 2x^{-1}$

Ⓑ $-\frac{12}{x^4}$ $\frac{d^2y}{dx^2} = -2x^{-2}$

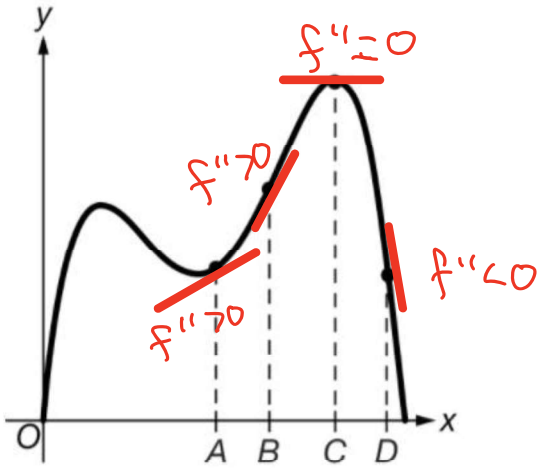
Ⓒ $\frac{16}{x^4}$ $\frac{d^3y}{dx^3} = +4x^{-3}$

Ⓓ $\frac{48}{x^5}$ $\frac{d^4y}{dx^4} = -12x^{-4}$

Unit 3 Progress Check MC Online

Scratch paper

Question 17



Graph of f'

The figure above shows the graph of f', the derivative of the function f. At which of the four indicated values of x is $f''(x)$ greatest? ie., when is the slope of f' the greatest?

(A) A

(B) B

(C) C

(D) D

Question 18

Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 3$ and $\frac{dy}{dx} = 4\sqrt{y^2 + 7x^2}$. What is the value of $\frac{d^2y}{dx^2}$ at $x = 1$?

(A) 10

(B) 23

(C) 55

(D) 160

$$\frac{d^2y}{dx^2} = 4 \cdot \frac{1}{2}\left(y^2+7x^2\right)^{-\frac{1}{2}} \cdot \left(2y\frac{dy}{dx} + 14x\right)$$

$$\frac{d^2y}{dx^2} = \frac{2\left(2y\frac{dy}{dx} + 14x\right)}{\sqrt{y^2+7x^2}}$$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = 4\sqrt{3^2+7(1)^2} = 4\sqrt{16} = 16$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,3)} = \frac{2\left(2(3)(16)+14(1)\right)}{\sqrt{3^2+7\cdot 1^2}} = \frac{2(110)}{\sqrt{16}} = \frac{2\cdot 110}{4} = 55$$