Skill Builder: Topic 6.2 – Approximating Areas with Riemann Sums

Answer each of the following problems. Be sure to show all necessary work.

1.) Consider the function $f(x) = -\frac{1}{2}x^2 + 8$ on the interval [0, 4]. Find approximations of the area lying between $f(x) = -\frac{1}{2}x^2 + 8$, the x-axis, $x = 0$, and $x = 4$ using the given number of n sub-intervals. Draw in the representative rectangles for each approximation.

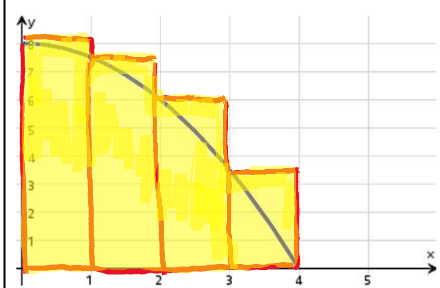
$$\boldsymbol{f(0) = 8, f(1) = 7.5, f(2) = 6, f(3) = 3.5, f(4) = 0}$$

Right Side Endpoints (Rectangle) $\boldsymbol{n = 4}$

$$A_R \approx \frac{4-0}{4}\left(f(1) + f(2) + f(3) + f(4)\right)$$

$$\approx \left(\frac{15}{2} + 6 + \frac{7}{2} + 0\right) = \frac{15+12+7}{2} = \frac{34}{2} = 17$$

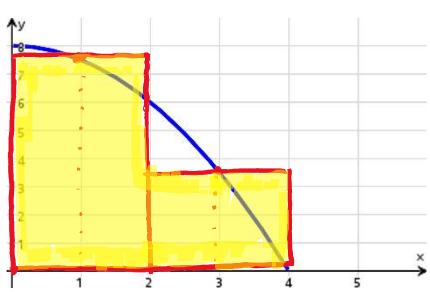
Left Side Endpoints (Rectangle) $\boldsymbol{n = 4}$



$$A_L \approx \frac{4-0}{4}\left(f(0) + f(1) + f(2) + f(3)\right)$$

$$\approx \left(8 + \frac{15}{2} + 6 + \frac{7}{2}\right) = \frac{16+15+12+7}{2} = \frac{50}{2} = 25$$

Midpoints (Rectangle) $\boldsymbol{n = 2}$



$$A_M \approx \frac{4-0}{2}\left(f(1) + f(3)\right)$$

$$\approx 2 \cdot \left(\frac{15}{2} + \frac{7}{2}\right) = 2 \cdot \frac{22}{2} = 22$$

Trapezoids $\boldsymbol{n = 4}$

$$A_T \approx \frac{4-0}{2 \cdot 4}\left(f(0) + 2 \cdot f(1) + 2 \cdot f(2) + 2 \cdot f(3) + f(4)\right)$$

$$\approx \frac{1}{2}\left(8 + 2 \cdot \frac{15}{2} + 2 \cdot 6 + 2 \cdot \frac{7}{2} + 0\right) \approx \frac{1}{2}\left(8 + 15 + 12 + 7\right)$$

$$\approx \frac{1}{2}(42) = 21$$

2.) Victor the custodian was withdrawing some cash from a local ATM when the machine malfunctioned and money began to fly out of the machine. The rate, in dollars per second, that money is coming out of the ATM is modeled by the function $M'(t)$, where t is measured in seconds. Selected values for $M'(t)$ are shown in the table below.

t	0	3	6	8	10
$M'(t)$	87	105	50	25	5

a.) Use a Left Riemann Sum with 4 subintervals indicated in the table to approximate the area under the curve $M'(t)$ and above the t-axis from $t = 0$ to $t = 10$.

$$A_L \approx 3(87)+3(105)+2(50)+2(25)$$
$$\approx 261+315+100+50$$
$$\approx 726$$

b.) Use a Right Riemann Sum with 4 subintervals indicated in the table to approximate the area under the curve $M'(t)$ and above the t-axis from $t = 0$ to $t = 10$.

$$A_R \approx 3(105)+3(50)+2(25)+2(5)$$
$$\approx 315+150+50+10$$
$$\approx 525$$

c.) Use a Midpoint Riemann Sum with 2 subintervals indicated in the table to approximate the area under the curve $M'(t)$ and above the t-axis from $t = 0$ to $t = 10$.

$$A_M \approx 6(105)+4(25)$$
$$\approx 630+100$$
$$\approx 730$$

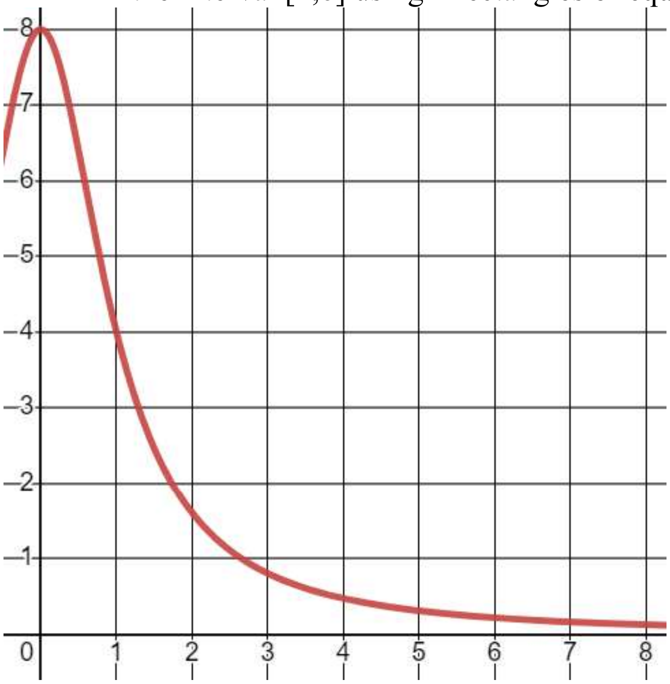
d.) Use a Trapezoidal Sum with 4 subintervals indicated in the table to approximate the area under the curve $M'(t)$ and above the t-axis from $t = 0$ to $t = 10$.

$$A_T \approx \frac{1}{2}(87+105)\cdot 3+\frac{1}{2}(105+50)\cdot 3+\frac{1}{2}(50+25)\cdot 2+\frac{1}{2}(25+5)\cdot 2$$
$$\approx \frac{1}{2}(192)(3)+\frac{1}{2}(155)(3)+\frac{1}{2}(75)(2)+\frac{1}{2}(30)(2)$$
$$\approx \frac{1}{2}(576+465+150+60)=\frac{1}{2}(1251)=625.5$$

e.) Using correct units, what does each approximation mean within the context of the problem?

Each approximation represents the amount of money in dollars that flew out of the ATM during the time interval $t = 0$ to $t = 10$ seconds.

3.) Using the Midpoint Rule to approximate the area under $f(x)=\dfrac{8}{x^2+1}$ and above the x-axis on the interval [2,6] using 4 rectangles of equal widths.



$$A_M \approx 1\cdot\left[f\left(\frac{5}{2}\right)+f\left(\frac{7}{2}\right)+f\left(\frac{9}{2}\right)+f\left(\frac{11}{2}\right)\right]$$

$$\approx \frac{8}{\left(\frac{5}{2}\right)^2+1}+\frac{8}{\left(\frac{7}{2}\right)^2+1}+\frac{8}{\left(\frac{9}{2}\right)^2+1}+\frac{8}{\left(\frac{11}{2}\right)^2+1}$$

$$\approx 2.33969$$

$f(x):=\dfrac{8}{x^2+1}$	*Done*
$f(x)\vert x=\left\{\dfrac{5}{2},\dfrac{7}{2},\dfrac{9}{2},\dfrac{11}{2}\right\}$	$\left\{\dfrac{32}{29},\dfrac{32}{53},\dfrac{32}{85},\dfrac{32}{125}\right\}$
$\dfrac{32}{29}+\dfrac{32}{53}+\dfrac{32}{85}+\dfrac{32}{125}$	$\dfrac{7641728}{3266125}$
$\dfrac{32}{29}+\dfrac{32}{53}+\dfrac{32}{85}+\dfrac{32}{125}$	2.33969

4.)

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function, f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values or r are given in the table above.

The total mass, in milligrams, of bacteria in the petri dish can be given by finding the area under the curve $r \cdot f(r)$ and multiplying by 2π. Approximate this mass on the interval $0 \le r \le 4$ using a right Reimann sum with the four subintervals indicated by the data in the table.

Let A_R = area under the curve $r \cdot f(r)$ on the interval $0 \le r \le 4$ using four right Reimann sums

$$A_R = \underbrace{1}_{\text{value of } r} \cdot \underbrace{1}_{\text{width}} \cdot \underbrace{f(1)}_{\text{height}} + \underbrace{2}_{\text{value of } r} \cdot \underbrace{1}_{\text{width}} \cdot \underbrace{f(2)}_{\text{height}} + \underbrace{2.5}_{\text{value of } r} \cdot \underbrace{0.5}_{\text{width}} \cdot \underbrace{f(2.5)}_{\text{height}} + \underbrace{4}_{\text{value of } r} \cdot \underbrace{1.5}_{\text{width}} \cdot \underbrace{f(4)}_{\text{height}}$$

$$= 1 \cdot 1 \cdot 2 + 2 \cdot 1 \cdot 6 + 2.5 \cdot 0.5 \cdot 10 + 4 \cdot 1.5 \cdot 18$$

$$= 2 + 12 + 12.5 + 108$$

$$= 134.5$$

Mass of bacteria population $\approx 2\pi(134.5) = 269\pi$ mg