Enrichment: Topic 6.3 – Riemann Sums, Summation Notation and Definite Integral Notation (Circuit)

Directions: Beginning in the first cell, find the answer. Hunt for that answer elsewhere in the document, mark that cell #2 and work the problem in that box. Process in this manner until you complete the circuit. You must show all your work. No calculators may be used on this circuit.
For each problem, use a right endpoint Riemann sum with equal partitions.

1

Ans: $\int_3^5 \sqrt{x^3+1}\,dx$

Find a limit expression equal to $\int_0^4 \left(x^2+1\right)dx$.

The difference between a and b could be $4-0=4$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=x^2+1$, a possible length could be

$\left(\frac{4}{n}k\right)^2+1$ or $\frac{16k^2}{n^2}+1 \rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2}+1\right)\left(\frac{4}{n}\right)$

7

Ans: $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(\frac{8}{n}k+1\right)\left(\frac{4}{n}\right)$

Find a limit expression equal to $\int_3^7 \left(x^2-8\right)dx$.

The difference between a and b could be $7-3=4$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=x^2-8$, a possible length could be

$\left(\frac{4}{n}k+3\right)^2-8$ or $\frac{16k^2}{n^2}+\frac{12k}{n}+9-8$

$\rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2}+\frac{12k}{n}+\frac{12k}{n}+1\right)\left(\frac{4}{n}\right)$

12

Ans: $\int_2^4 \left(x^2+1\right)dx$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{2}{n}k\right)^3+3\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\left(\left(\frac{2}{n}k\right)^3+3\right)$,

it is possible that $a=0$ and $f(x)=x^3+3$ or

This leads us to $\int_0^2 \left(x^3+3\right)dx$.

10

Ans: $\int_0^2 \left(x^3+2\right)dx$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\left(\frac{2}{n}k+3\right)+1}\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\sqrt{\left(\frac{2}{n}k+3\right)+1}$,

it is possible that $a=3$ and $f(x)=\sqrt{x+1}$.

This leads us to $\int_3^5 \sqrt{x+1}\,dx$.

 # 15

Ans: $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\ln\left(\frac{4}{n}k+1\right)\left(\frac{4}{n}\right)$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{2}{n}k+3\right)+1\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\left(\left(\frac{2}{n}k+3\right)+1\right)$,

it is possible that $a=3$ and $f(x)=x+1$.

This leads us to $\int_3^5 (x+1)\,dx$.

 # 4

Ans: $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\sqrt{\frac{16}{n}k+9}\right)\left(\frac{4}{n}\right)$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{2}{n}k\right)^2+3\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\left(\left(\frac{2}{n}k\right)^2+3\right)$,

it is possible that $a=3$ and $f(x)=x^2$.

This leads us to $\int_3^5 x^2\,dx$ which is not an option in the circuit.

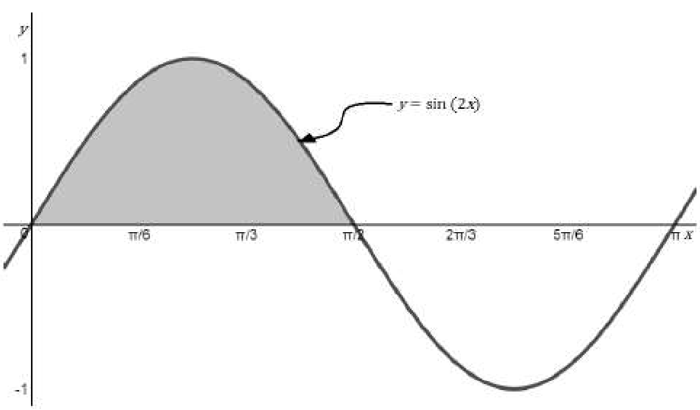
So, it is more likely $a=0$ and $f(x)=x^2+3$.

This leads us to $\int_0^2 \left(x^2+3\right)dx$.

 # 8

Ans: $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\frac{16}{n^2}k^2+\frac{24}{n}k+1\right)\left(\frac{4}{n}\right)$

Find a limit expression that is equal to the area of the shaded region shown below.



The values of a and b are $a=0$ and $b=\pi/2$.

Thus a possible width could be $\frac{\pi/2}{n}=\frac{\pi}{2n}$.

Since $f(x)=\sin(2x)$, a possible length could be

$\sin\left(2\cdot\frac{\pi}{2n}k\right) \rightarrow \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\left(\sin\left(\frac{\pi k}{n}\right)\right)\left(\frac{\pi}{2n}\right)\right)$.

 # 2

Ans: $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\frac{16}{n^2}k^2+1\right)\left(\frac{4}{n}\right)$

Find a limit expression equal to $\int_0^{\pi/2}\cos(2x)\,dx$.

The difference between a and b could be $\frac{\pi}{2}-0=\frac{\pi}{2}$.

Thus a possible width could be $\frac{\pi/2}{n}$ or $\frac{\pi}{2n}$.

Since $f(x)=\cos(2x)$, a possible length could be

$\cos\left(2\cdot\frac{\pi/2}{n}k\right)$ or $\cos\left(\frac{\pi k}{n}\right)$

$\rightarrow \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\cos\left(\frac{\pi}{n}k\right)\right)\left(\frac{\pi}{2n}\right)$.

3

Ans: $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\cos\left(\frac{\pi}{n}k\right)\right)\left(\frac{\pi}{2n}\right)$

Find a limit expression equal to $\int_3^7 \sqrt{4x-3}\,dx$.

The difference between a and b could be $7-3=4$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=\sqrt{4x-3}$ and $a=3$, a possible length could be

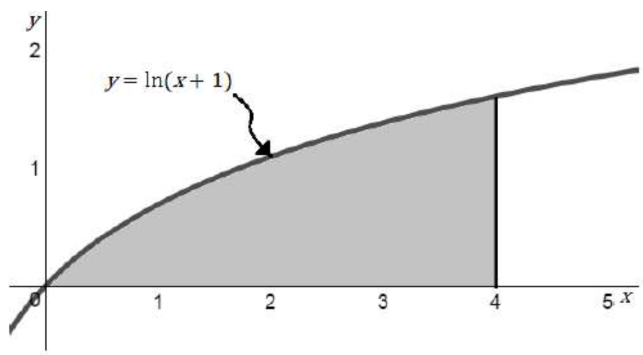
$\sqrt{4\left(3+\frac{4k}{n}\right)-3}$ or $\sqrt{12+\frac{16k}{n}-3}=\sqrt{\frac{16k}{n}+9}$

$\rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\frac{16}{n}k+9}\right)\left(\frac{4}{n}\right)$

14

Ans: $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\frac{4}{n}k+2}\right)\left(\frac{4}{n}\right)$

Find a limit expression that is equal to the area of the shaded region shown below.



The values of a and b are $a=0$ and $b=4$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=\ln(x+1)$, a possible length could be

$\ln\left(\frac{4k}{n}+1\right) \rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(\ln\left(\frac{4k}{n}+1\right)\right)\left(\frac{4}{n}\right)$.

9

Ans: $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sin\left(\frac{\pi}{n}k\right)\right)\left(\frac{\pi}{2n}\right)$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{2}{n}k\right)^3+2\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\left(\left(\frac{2}{n}k\right)^3+2\right)$,

it is possible that $a=0$ and $f(x)=x^3+2$ or

$a=2$ and $f(x)=x^3$ (which will not be an option).

This leads us to $\int_0^2\left(x^3+2\right)dx$.

16

Ans: $\int_3^5 (x+1)\,dx$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\left(\frac{2}{n}k+3\right)^3+1}\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\sqrt{\left(\left(\frac{2}{n}k+3\right)^3+1\right)}$,

it is possible that $a=3$ and $f(x)=\sqrt{x^3+1}$.

This leads us to $\int_3^5\sqrt{x^3+1}\,dx$.

 # 11

Ans: $\int_3^5 \sqrt{x+1}\,dx$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{2}{n}k+2\right)^2+1\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\left(\left(\frac{2}{n}k+2\right)^2+1\right)$,

it is possible that $a=2$ and $f(x)=x^2+1$ or

This leads us to $\int_2^4 \left(x^2+1\right)dx$.

 # 5

Ans: $\int_0^2 \left(x^2+3\right)dx$

Find an integral expression equal to

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\left(\frac{2}{n}k+3\right)^2+1}\right)\left(\frac{2}{n}\right).$$

The width of the region is $b-a=2$.

Since we are given a length of $\sqrt{\left(\left(\frac{2}{n}k+3\right)^2+1\right)}$,

it is possible that $a=3$ and $f(x)=\sqrt{x^2+1}$.

This leads us to $\int_3^5 \sqrt{x^2+1}\,dx$.

 # 6

Ans: $\int_3^5 \sqrt{x^2+1}\,dx$

Find a limit expression equal to $\int_1^5 \ln(2x-1)dx$.

The difference between a and b could be $5-1=4$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=\ln\left(2x-1\right)$ and $a=1$, a possible length could be

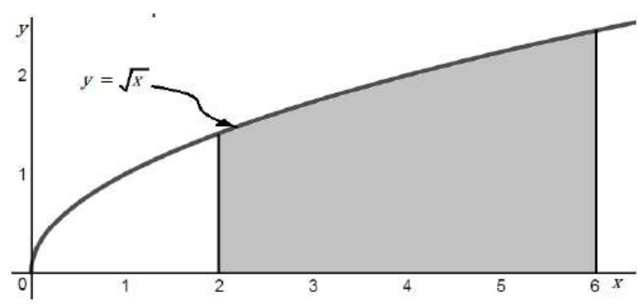
$\ln\left(2\cdot\frac{4k}{n}+1\right)$ or $\ln\left(\frac{8k}{n}+1\right)$

$\to \lim_{n\to\infty}\sum_{k=1}^{n}\left(\ln\left(\frac{8k}{n}+1\right)\right)\left(\frac{4}{n}\right)$

 # 13

Ans: $\int_0^2 \left(x^3+3\right)dx$

Find a limit expression that is equal to the area of the shaded region shown below.



The values of a and b are $a=2$ and $b=6$.

Thus a possible width could be $\frac{4}{n}$.

Since $f(x)=\sqrt{x}$, a possible length could be

$\sqrt{\frac{4k}{n}+2} \to \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\sqrt{\frac{4k}{n}+2}\right)\left(\frac{4}{n}\right)\right).$