Skill Builder: Topic 6.6 – Applying Properties of Definite Integrals

1. Suppose that f and g are continuous functions and that $\int_1^2 f(x)\,dx = -4$, $\int_1^5 f(x)\,dx = 6$, and $\int_1^5 g(x)\,dx = 8$. Find each of the following

a. $\int_2^2 g(x)\,dx$ $= 0$	b. $\int_5^1 g(x)\,dx$ $= -\int_1^5 g(x)\,dx = -8$	c. $\int_1^2 3f(x)\,dx$ $= 3\cdot(-4) = -12$
d. $\int_2^5 f(x)\,dx$ $= \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 6-(-4) = 10$	e. $\int_1^5 [f(x)-g(x)]\,dx$ $= 6-8 = -2$	f. $\int_1^5 [4f(x)-g(x)]\,dx$ $= 4(6)-8 = 24-8 = 16$

2. Suppose that f is a continuous function such that $\int_0^3 f(t)\,dt = 3$ and $\int_0^4 f(t)\,dt = 7$.

Find each of the following.

a. $\int_3^4 f(t)\,dt$ $= \int_0^4 f(t)\,dt - \int_0^3 f(t)\,dt = 7-3 = 4$	b. $\int_4^3 f(t)\,dt$ $= -\int_3^4 f(t)\,dt = -4$

3. If $\int_3^7 h(x)\,dx = 5$ and $\int_3^7 k(x)\,dx = 3$, which one of the following statements is NOT true?

(A) $\int_3^7 h(x)k(x)\,dx = 15$ (B) $\int_3^7 [h(x)+k(x)]\,dx = 8$ (C) $\int_3^7 2h(x)\,dx = 10$

(D) $\int_3^7 [h(x)-k(x)]\,dx = 2$ (E) $\int_7^3 [k(x)-h(x)]\,dx = 2$ (F) $\int_3^3 [h(x)+k(x)]\,dx = 0$

4. Suppose that g is a continuous function such that $\int_{-2}^{1} g(x)\,dx = 2$ and $\int_{1}^{3} g(x)\,dt = -6$.

Find $\int_{3}^{-2} g(x)\,dx$.

$= -\int_{-2}^{3} g(x)\,dx = -\left(\int_{-2}^{1} g(x)\,dx + \int_{1}^{3} g(x)\,dx\right)$

$= -(2 + (-6)) = -(-4) = 4$

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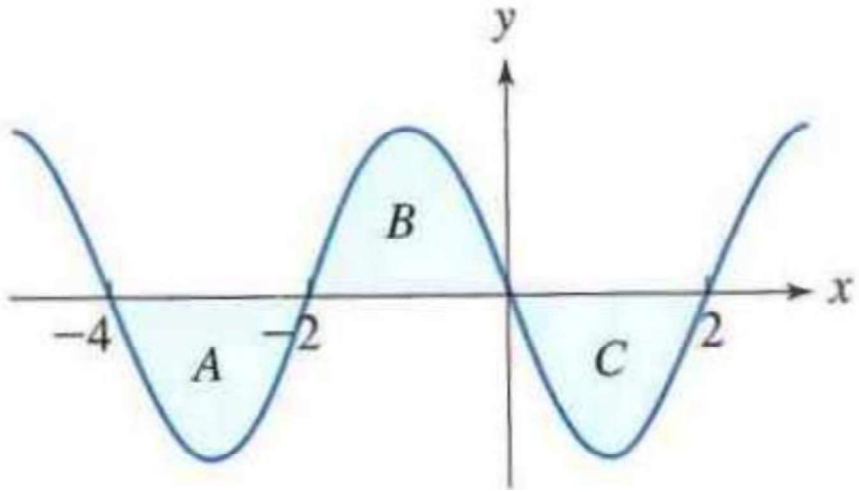
5. Each of the regions in the figure bounded by the graph of $f(x)$ and the x-axis has area 3.



Find each of the following.

a. $\int_{-4}^{2} [4f(x) - 5]\,dx$

$= 4(-3 + 3 - 3) - 5(6)$

$= -12 - 30 = -42$

b. $\int_{-4}^{2} |f(x)|\,dx$

$= 3 + 3 + 3 = 9$

c. $\int_{2}^{-4} f(x)\,dx$

$= -\int_{-4}^{2} f(x)\,dx$

$= -(-3 + 3 - 3) = 3$

d. $\int_{-2}^{2} f(|x|)\,dx$

$= -3 + -3$

$= -6$

e. $\left|\int_{-4}^{2} f(x)\,dx\right|$

$= |-3 + 3 - 3| = 3$

f. $\int_{-2}^{4} f(-x)\,dx$

$= -3 + 3 - 3$

$= -3$