Skill Builder: Topic 6.7 – The Fundamental Theorem of Calculus and Definite Integrals (Big 10)

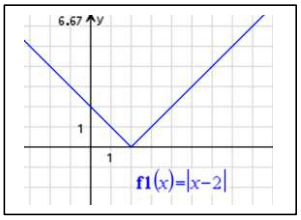
Complete the ten problems below. Once you complete each problem, cross off the appropriate value in the box below according to the intructions. If done correctly, you will have all 10 numbers crossed off with no repeats. Express all fractional answers as improper fractions.

0	1	2	3	4	5	6	7	8	9

Problem	Instructions
1. $\int_0^1 3x\,dx = \frac{3}{2}x^2\Big\|_0^1 = \frac{3}{2}(1)^2 - \frac{3}{2}(0)^2 = \frac{3}{2}$	Cross off the value of the numerator of your answer. Cross off 3.
2. $\int_{-2}^{3}(x-5)\,dx =$ $\left(\frac{1}{2}x^2-5x\right)\Big\|_{-2}^{3} = \frac{1}{2}(3)^2-5(3)-\left[\frac{1}{2}(-2)^2-5(-2)\right]$ $= \frac{9}{2}-15-(2+10) = \frac{9}{2}-27 = \frac{9}{2}-\frac{54}{2} = -\frac{45}{2}$	Cross off the value that is equivalent to the sum of the positive digits of the numerator of your answer. $4+5=9$ Cross off 9.
3. Given $g(x)=\int_{\pi}^{x}\frac{1}{1+t^4}\,dt$, find $g'(2)$. $g'(x)=\frac{1}{1+x^4}$; $g'(2)=\frac{1}{1+2^4}=\frac{1}{17}$	Cross off the units (ones) digit in denominator of your answer. Cross off 7.
4. $\int_{-2}^{-1}\left(x-\frac{1}{x^2}\right)dx$ $=\frac{1}{2}x^2-\left(-\frac{1}{x}\right)\Big\|_{-2}^{-1} = \frac{1}{2}(-1)^2-\frac{1}{1}-\left(\frac{1}{2}(-2)^2-\frac{1}{2}\right)$ $= \frac{1}{2}-1-2+\frac{1}{2} = -2$	Cross off the absolute value of your answer. Cross off 2.
5. $\int_1^9\left(\frac{x-2}{\sqrt{x}}\right)dx$ $= \int_1^9\left(\frac{x}{\sqrt{x}}-\frac{2}{\sqrt{x}}\right)dx = \int_1^9\left(x^{1/2}-2x^{-1/2}\right)dx$ $= \frac{2}{3}x^{3/2}-4x^{1/2}\Big\|_1^9 = \frac{2}{3}\left(\sqrt{9}\right)^3-4\sqrt{9}-\left(\frac{2}{3}\left(\sqrt{1}\right)^3-4\sqrt{1}\right)$ $= \frac{2}{3}\cdot 27-12-\frac{2}{3}+4 = 18-12-\frac{2}{3}+4 = 10-\frac{2}{3} = \frac{30}{3}-\frac{2}{3} = \frac{28}{3}$	Cross off the units (ones) digit in numerator of your answer. Cross off 8.

6. $\int_0^3 |x-2|\,dx$ (Hint: Think about the area under the curve.)

$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) = 2 + \frac{1}{2} = \frac{5}{2}$



Cross off the value of the numerator of your answer.

Cross off 5.

7. Given $f(x) = \int_{-1}^{x^2} \sqrt{t^3+3}\,dt$, find $f'(1)$.

$f'(x) = \left(\sqrt{x^6+3}\right)(2x)$

$f'(1) = \left(\sqrt{(1)^6+3}\right)\cdot(2\cdot 1) = 2\sqrt{4} = 2\cdot 2 = 4$

Cross off the value of your answer.

Cross off 4.

8. $\int_0^{\pi/3} \sec\theta\tan\theta\,d\theta$

$= \sec\theta\Big|_0^{\pi/3} = \sec\left(\frac{\pi}{3}\right) - \sec(0)$

$= \frac{1}{\cos\left(\frac{\pi}{3}\right)} - \frac{1}{\cos(0)} = \frac{1}{\frac{1}{2}} - \frac{1}{1} = 2 - 1 = 1$

Cross off the value of your answer.

Cross off 1.

9. Which of the following definite integrals is undefined?

(5) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$ **(6)** $\int_0^2 \frac{1}{x^2}\,dx$ **(7)** $\int_{1/2}^{-1/2} \frac{1}{\sqrt{1-x^2}}\,dx$ **(8)** $\int_1^2 \frac{1}{(x-3)^2}\,dx$

We cannot solve (6). This requires a technique taught Calculus BC. The reason is because the lower boundary of 0 will cause the expression in the integrand to be undefined.

Cross off the value that corresponds to which definite integral was undefined.

Cross off 6.

10. Let $f(x)$ be an odd function that is integrable on the interval $-a \le x \le a$, what is $\int_{-a}^{a} f(x)\,dx$?

Whenever an odd function is integrated symmetrically across the origin the two areas obtained on either side will be the same size but with opposite signs. Recall that an odd function is symmetrical with respect to the origin. This will result in a net area of 0.

Cross off the value of your answer.

Cross off 0.