Skill Builder: Topic 6.8 – Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation (Circuit)

Begin in the first cell marked #1 and find the antiderivative of each given function. To advance in the circuit, search for your answer and mark that cell #2. Continue in this manner until you complete the circuit. Show all pertinent work.

 # 1

Ans: $-\sin x+\frac{3x^2}{2}+3x+1$

$\int -9\,dx$

$-9x+C$

 # 5

Ans: $-\frac{7}{x}+C$

$\int \frac{7}{x}\,dx$

$=7\cdot\int\frac{1}{x}dx$

$=7\ln|x|+C$ or $\ln(|x|)^7+C$

 # 11

Ans: $\frac{3x^{4/3}}{2}+C$

$\int\left(x^{3/4}-\frac{1}{x^{3/4}}\right)dx$

$=\int\left(x^{3/4}-x^{-3/4}\right)dx$

$=\frac{x^{7/4}}{\frac{7}{4}}-\frac{x^{1/4}}{\frac{1}{4}}+C$

$=\frac{4}{7}x^{7/4}-4x^{1/4}+C$

 # 20

Ans: $x+C$

$\int\frac{\sin x}{1-\sin^2 x}dx$

$=\int\frac{\sin x}{\cos^2 x}dx$

$=\int\left(\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\right)dx$

$=\int(\tan x\cdot\sec x)\,dx$

$=\sec x+C$

 # 26

Ans: $3e^x+\ln|x|+\frac{1}{x}+C$

Given $f'''(x)=\cos x$, $f(0)=1$, $f'(0)=2$, and $f''(0)=3$, find $f(x)$.

$\int f'''(x)\,dx=f''(x)=\int\cos x\,dx=\sin x+C_1$

$f''(0)=3\rightarrow 3=\sin(0)+C_1 \rightarrow C_1=3$

$\int f''(x)\,dx=f'(x)=\int(\sin x+3)\,dx=-\cos x+3x+C_2$

$f'(0)=2\rightarrow 2=-\cos(0)+3(0)+C_2 \rightarrow C_2=3$

$\int f'(x)\,dx=\int(-\cos x+3x+3)\,dx=-\sin x+\frac{3}{2}x^2+3x+C_3$

$f(0)=1\rightarrow 1=-\sin(0)+\frac{3}{2}(0)^2+3(0)+C_3 \rightarrow C_3=1$

$\therefore f(x)=-\sin x+\frac{3}{2}x^2+3x+1$

 # 8

Ans: $\frac{9^x}{\ln(9)}+C$

$\int\left(2-\frac{1}{x^5}+\frac{7}{x^3}\right)dx$

$=\int\left(2-x^{-5}+7x^{-3}\right)dx$

$=2\cdot x-\frac{x^{-4}}{-4}+7\cdot\frac{x^{-2}}{-2}+C$

$=2x+\frac{1}{4x^4}-\frac{7}{2x^2}+C$

 # 22 **Ans:** $x^2+2x-10$

Given $f''(x)=2x$, $f'(2)=-1$, and $f(3)=1$, find $f(x)$.

$\int f''(x)\,dx = f'(x) = \int 2x\,dx = x^2 + C_1$

$f'(2)=-1 \to -1 = 2^2 + C_1 \to C_1 = -5$

$\int f'(x)\,dx = f(x) = \int \left(x^2-5\right)dx = \frac{1}{3}x^3 - 5x + C_2$

$f(3)=1 \to 1 = \frac{1}{3}\cdot 3^3 - 5(3) + C_2 \to C_2 = 7$

$\therefore\ f(x) = \frac{1}{3}x^3 - 5x + 7$

 # 18 **Ans:** $-\frac{1}{x}+\cos x + C$

$\int \left(\sec^2 x + \cos x + 1\right)dx$

$= \tan x + \sin x + x + C$

 # 4 **Ans:** x^2+6x+C

$\int \frac{7}{x^2}\,dx$

$= \int 7x^{-2}\,dx$

$= 7\cdot\frac{x^{-1}}{-1} + C$

$= -\frac{7}{x} + C$

 # 2 **Ans:** $-9x+C$

$\int -5x\,dx$

$= -5\cdot\frac{x^2}{2} + C$

$= -\frac{5}{2}x^2 + C$

 # 14 **Ans:** $\frac{x^3}{3} - 5x^2 + 25x + C$

$\int \frac{x^3-4x-1}{2x^3}\,dx$

$= \int\left(\frac{x^3}{2x^3} - \frac{4x}{2x^3} - \frac{1}{2x^3}\right)dx$

$= \int\left(\frac{1}{2} - 2x^{-2} - \frac{1}{2}x^{-3}\right)dx$

$= \frac{1}{2}x - 2\cdot\frac{x^{-1}}{-1} - \frac{1}{2}\cdot\frac{x^{-2}}{-2} + C$

$= \frac{1}{2}x + \frac{2}{x} + \frac{1}{4x^2} + C$

 # 25 **Ans:** $-\cos x + 2x - (2+2\pi)$

$\int\left(3e^x + \frac{1}{x} - \frac{1}{x^2}\right)dx$

$= \int\left(3e^x + \frac{1}{x} - x^{-2}\right)dx$

$= 3e^x + \ln|x| - \frac{x^{-1}}{-1} + C$

$= 3e^x + \ln|x| + \frac{1}{x} + C$

 # 12

Ans: $\dfrac{4x^{7/4}}{7} - 4x^{1/4} + C$

$\int 3\sqrt[3]{x^2}$

$= 3\int x^{2/3}\,dx$

$= 3\cdot\dfrac{x^{5/3}}{\frac{5}{3}} + C$

$= \dfrac{9}{5}x^{5/3} + C$

 # 9

Ans: $2x + \dfrac{1}{4x^4} - \dfrac{7}{2x^2} + C$

$\int 5\sqrt{x}\,dx$

$= 5\int x^{1/2}\,dx$

$= 5\cdot\dfrac{x^{3/2}}{\frac{3}{2}} + C$

$= \dfrac{10}{3}x^{3/2} + C$

 # 23

Ans: $\dfrac{x^3}{3} - 5x + 7$

Given $f''(x) = \dfrac{1}{x^{3/2}}$, $f'(4) = 2$, and $f(0) = 1$,

find $f(x)$.

$\int f''(x)\,dx = f'(x) = \int x^{-3/2}\,dx = -2x^{-1/2} + C_1 = \dfrac{-2}{\sqrt{x}} + C_1$

$f'(4) = 2 \rightarrow 2 = \dfrac{-2}{\sqrt{4}} + C_1 \rightarrow C_1 = 3$

$\int f'(x)\,dx = f(x) = \int\left(-2x^{-1/2} + 3\right)dx = -2\cdot\dfrac{x^{1/2}}{\frac{1}{2}} + 3x + C_2$

$f(0) = 1 \rightarrow 1 = -4\sqrt{0} + 2(0) + C_2 \rightarrow C_2 = 1$

$\therefore\ f(x) = -4\sqrt{x} + 3x + 1$

 # 15

Ans: $\dfrac{1}{2}\left(x + \dfrac{4}{x} + \dfrac{1}{2x^2}\right) + C$

$\int x^2(3+x)^2\,dx$

$= \int\left(x^2\left(9 + 6x + x^2\right)\right)dx$

$= \int\left(x^4 + 6x^3 + 9x^2\right)dx$

$= \dfrac{x^5}{5} + 6\cdot\dfrac{x^4}{4} + 9\cdot\dfrac{x^3}{3} + C$

$= \dfrac{1}{5}x^5 + \dfrac{3}{2}x^4 + 3x^3 + C$

 # 6

Ans: $\ln\left(|x|\right)^7 + C$

$\int\left(\dfrac{2}{3}x^5 - \dfrac{5}{2}x + \dfrac{1}{2}\right)dx$

$= \dfrac{2}{3}\cdot\dfrac{x^6}{6} - \dfrac{5}{2}\cdot\dfrac{x^2}{2} + \dfrac{1}{2}\cdot x + C$

$= \dfrac{1}{9}x^6 - \dfrac{5}{4}x^2 + \dfrac{1}{2}x + C$

 # 21

Ans: $\sec x + C$

Given $f''(x) = 2$, $f'(1) = 4$, and $f(2) = -2$,

find $f(x)$.

$\int f''(x)\,dx = f'(x) = \int 2\,dx = 2x + C_1$

$f'(1) = 4 \rightarrow 4 = 2(1) + C_1 \rightarrow C_1 = 2$

$\int f'(x)\,dx = f(x) = \int(2x + 2)\,dx = x^2 + 2x + C_2$

$f(2) = -2 \rightarrow -2 = 2^2 + 2(2) + C_2 \rightarrow C_2 = -10$

$\therefore\ f(x) = x^2 + 2x - 10$

 # 17

Ans: $\frac{18x^{5/2}}{5} - 8x^{3/2} + 8\sqrt{x} + C$

$$\int \left(\frac{1}{x^2} - \sin x \right) dx$$

$$= \int \left(x^{-2} - \sin x \right) dx$$
$$= \frac{x^{-1}}{-1} - \cos x + C$$
$$= -\frac{1}{x} - \cos x + C$$

 # 13

Ans: $\frac{9x^{5/3}}{5} + C$

$$\int (x-5)^2\, dx$$

$$= \int \left(x^2 - 10x + 25 \right) dx$$
$$= \frac{x^3}{3} - 10 \cdot \frac{x^2}{2} + 25x + C$$
$$= \frac{1}{3}x^3 - 5x^2 + 25x + C$$

 # 10

Ans: $\frac{10x^{3/2}}{3} + C$

$$\int 2\sqrt[3]{x}\, dx$$

$$= 2\int x^{1/3}\, dx$$
$$= 2 \cdot \frac{x^{4/3}}{\frac{4}{3}} + C$$
$$= \frac{3}{2}x^{4/3} + C$$

 # 24

Ans: $-4\sqrt{x} + 3x + 1$

Given $f''(x) = \cos x$, $f'(\pi) = 2$, and $f(\pi) = -1$, find $f(x)$.

$$\int f''(x)\, dx = f'(x) = \int \cos x dx = \sin x + C_1$$
$$f'(\pi) = 2 \rightarrow 2 = \sin \pi + C_1 \rightarrow C_1 = 2$$
$$\int f'(x)\, dx = f(x) = \int \left(\sin x + 2 \right) dx = -\cos x + 2x + C_2$$
$$f(\pi) = -1 \rightarrow -1 = -\cos \pi + 2\pi + C_2 \rightarrow C_2 = -2 - 2\pi$$
$$\therefore f(x) = -\cos x + 2x - 2 - 2\pi$$

 # 7

Ans: $\frac{x^6}{9} - \frac{5x^2}{4} + \frac{x}{2} + C$

$$\int 9^x\, dx$$

$$= \frac{1}{\ln 9} \cdot 9^x + C$$

 # 3

Ans: $-\frac{5x^2}{2} + C$

$$\int (6 + 2x)\, dx$$

$$= 6x + 2 \cdot \frac{x^2}{2} + C$$
$$= 6x + x^2 + C$$

19

Ans: $\tan x + \sin x + x + C$

$$\int \left(\sin^2 x + \cos^2 x\right) dx$$

$$= \int (1)\, dx$$
$$= x + C$$

16

Ans: $3x^3 + \frac{3x^4}{2} + \frac{x^5}{5} + C$

$$\int \frac{(3x-2)^2}{\sqrt{x}}\, dx$$

$$= \int \frac{9x^2 - 12x + 4}{x^{1/2}}\, dx$$
$$= \int \left(\frac{9x^2}{x^{1/2}} - \frac{12x}{x^{1/2}} + \frac{4}{x^{1/2}} \right) dx$$
$$= \int \left(9x^{3/2} - 12x^{1/2} + 4x^{-1/2} \right) dx$$
$$= 9 \cdot \frac{x^{5/2}}{\frac{5}{2}} - 12 \cdot \frac{x^{3/2}}{\frac{3}{2}} + 4 \cdot \frac{x^{1/2}}{\frac{1}{2}} + C$$
$$= \frac{18}{5} x^{5/2} - 8x^{3/2} + 8x^{1/2} + C$$