Skill Builder: Topic 6.9 – Integration Using Substitution (Circuit)

Begin in the first cell marked #1 and calculate the indefinite integral of each given function. To advance in the circuit, search for your answer and mark that cell #2. Continue in this manner until you complete the circuit. Show all pertinent work.

 # 1

Ans: $\frac{1}{2}\ln\left|x^2+8x-3\right|+C$

$\int e^{6x}\,dx$

$u=6x \rightarrow du=6\,dx \rightarrow dx=\frac{du}{6}$

$=\int e^u \frac{du}{6}=\frac{1}{6}e^u+C$

$=\frac{1}{6}e^{6x}+C$

 # 26

Ans: $-\frac{1}{4}\cos^4 x+C$

$\int (2x+3)^{11}\,dx$

$u=2x+3 \rightarrow du=2\,dx \rightarrow dx=\frac{du}{2}$

$=\int u^{11}\frac{du}{2}=\frac{1}{2}\int u^{11}\,du$

$=\frac{1}{2}\cdot\frac{1}{12}u^{12}+C$

$=\frac{1}{24}(2x+3)^{12}+C$

 # 16

Ans: $-\frac{1}{9}(\cos(6x))^{3/2}+C$

$\int \frac{x}{\sqrt[3]{x^2+1}}\,dx$

$u=x^2+1 \rightarrow du=2x\,dx \rightarrow dx=\frac{du}{2x}$

$=\int \frac{x}{\sqrt[3]{u}}\cdot\frac{du}{2x}=\int \frac{1}{2}u^{-1/3}\,du$

$=\frac{1}{2}\cdot\frac{u^{2/3}}{\frac{2}{3}}+C=\frac{3}{4}(x^2+1)^{2/3}+C$

 # 10

Ans: $\frac{5^{\sin x}}{\ln 5}+C$

$\int \sin x\cdot e^{\cos x}\,dx$

$u=\cos x \rightarrow du=-\sin x\,dx \rightarrow dx=\frac{du}{-\sin x}$

$=\int \sin x e^u \frac{du}{-\sin x}=-\int e^u\,du$

$=-e^u+C$

$=-e^{\cos x}+C$

 # 19

Ans: $-\frac{1}{4}\left(1+\frac{1}{x}\right)^4+C$

$\int 5(3-4x)^{2/3}\,dx$

$u=3-4x \rightarrow du=-4\,dx \rightarrow dx=\frac{du}{-4}$

$=\int 5u^{2/3}\frac{du}{-4}=-\frac{5}{4}\int u^{2/3}\,du$

$=-\frac{5}{4}\frac{u^{5/3}}{\frac{5}{3}}+C=-\frac{3}{4}(3-4x)^{5/3}+C$

 # 33

Ans: $\frac{3}{2}\arcsin\left(\frac{2x}{3}\right)+C$

$\int \frac{dx}{(8x-1)^3}$

$u=8x-1 \rightarrow du=8\,dx \rightarrow dx=\frac{du}{8}$

$=\int u^{-3}\frac{du}{8}=\frac{1}{8}\int u^{-3}\,du$

$=\frac{1}{8}\cdot\frac{u^{-2}}{-2}+C=-\frac{1}{16u^2}+C$

$=-\frac{1}{16(8x-1)^2}+C$

 # 2

Ans: $\frac{1}{6}e^{6x}+C$

$\int x\left(x^2+2\right)^6\,dx$

$u=x^2+2 \rightarrow du=2x\,dx \rightarrow dx=\dfrac{du}{2x}$

$=\int x(u)^6\dfrac{du}{2x}=\int \dfrac{1}{2}u^6 du$

$=\dfrac{1}{2\cdot 7}u^7+C=\dfrac{1}{14}(x^2+2)^7+C$

 # 30

Ans: $\frac{1}{6}(\ln x)^6+C$

$\int \dfrac{5}{x\ln x}dx$

$u=\ln x \rightarrow du=\dfrac{1}{x}dx \rightarrow dx=x\,du$

$=\int \dfrac{5}{x\cdot u}x\,du=\int \dfrac{5}{u}du$

$=5\ln|u|+C$

$=5\ln|\ln x|+C$

 # 18

Ans: $\frac{1}{2}x^2-3x-5\ln|x|+C$

$\int\left(1+\dfrac{1}{x}\right)^3\left(\dfrac{1}{x^2}\right)dx$

$u=1+\dfrac{1}{x} \rightarrow du=-\dfrac{1}{x^2}dx \rightarrow dx=-x^2\,du$

$=\int u^3\left(\dfrac{1}{x^2}\right)\left(-x^2\,du\right)=-\int u^3 du$

$=-\dfrac{1}{4}u^4+C=-\dfrac{1}{4}\left(1+\dfrac{1}{x}\right)^4+C$

 # 21

Ans: $-\frac{20}{27}\left(4-\frac{3}{5}x\right)^{3/2}+C$

$\int x^{1/3}\left(x^{4/3}+9\right)^8\,dx$

$u=x^{4/3}+9 \rightarrow du=\dfrac{4}{3}x^{1/3}\,dx \rightarrow dx=\dfrac{3}{4x^{1/3}}du$

$=\int x^{1/3}\cdot u^8\left(\dfrac{3}{4x^{1/3}}du\right)=\dfrac{3}{4}\int u^8 du$

$=\dfrac{3}{4}\cdot\dfrac{1}{9}u^9+C=\dfrac{1}{12}\left(x^{4/3}+9\right)^9+C$

 # 20

Ans: $-\frac{3}{4}(3-4x)^{5/3}+C$

$\dfrac{2}{3}\int\sqrt{4-\dfrac{3}{5}x}\,dx$

$u=4-\dfrac{3}{5}x \rightarrow du=-\dfrac{3}{5}dx \rightarrow dx=-\dfrac{5}{3}du$

$=\dfrac{2}{3}\int\sqrt{u}\left(-\dfrac{5}{3}du\right)=-\dfrac{10}{9}\int u^{1/2}du$

$=-\dfrac{10}{9}\dfrac{u^{3/2}}{\frac{3}{2}}+C=-\dfrac{20}{27}\left(4-\dfrac{3}{5}x\right)^{3/2}+C$

 # 8

Ans: $-\frac{1}{3}\ln\left|5-x^3\right|+C$

$\int\csc(2x)\,dx$

$u=2x \rightarrow du=2\,dx \rightarrow dx=\dfrac{du}{2}$

$=\int\csc u\dfrac{du}{2}=-\dfrac{1}{2}\ln|\csc u+\cot u|+C$

$=-\dfrac{1}{2}\ln|\csc 2x+\cot 2x|+C$

 # 28

Ans: $\ln\left|x^2-5x-1\right|+C$

$\int \sin 5x\,dx$

$u=5x \rightarrow du=5\,dx \rightarrow dx=\frac{du}{5}$

$=\int \sin u\cdot\frac{du}{5}=\frac{1}{5}\int \sin u\,du$

$=-\frac{1}{5}\cos u+C$

$=-\frac{1}{5}\cos 5x+C$

 # 13

Ans: $\frac{4}{9}\left(3x^3-1\right)^{3/2}+C$

$\int \sqrt{x+2}\,dx$

$u=x+2 \rightarrow du=dx$

$=\int u^{1/2}du$

$=\frac{u^{3/2}}{\frac{3}{2}}+C=\frac{2}{3}(x+2)^{3/2}+C$

 # 27

Ans: $\frac{1}{24}\left(2x+3\right)^{12}+C$

$\int \frac{2x-5}{x^2-5x-1}dx$

$u=x^2-5x-1 \rightarrow du=(2x-5)\,dx \rightarrow dx=\frac{du}{2x-5}$

$=\int \frac{2x-5}{u}\cdot\frac{du}{2x-5}=\int \frac{1}{u}du$

$=\ln|u|+C$

$=\ln\left|x^2-5x-1\right|+C$

 # 3

Ans: $\frac{1}{14}\left(x^2+2\right)^7+C$

$\int \sin 4x\cos 4x\,dx$

$u=\sin 4x \rightarrow du=4\cdot\cos 4x\,dx \rightarrow dx=\frac{du}{4\cos 4x}$

$=\int u\cdot\cos 4x\cdot\frac{du}{4\cos 4x}=\int \frac{1}{4}u\,du$

$=\frac{1}{4\cdot 2}u^2+C=\frac{1}{8}u^2+C=\frac{1}{8}\sin^2 4x+C$

 # 25

Ans: $-\frac{5}{3}\ln|3x-4|+C$

$\int \cos^3 x\sin x\,dx$

$u=\cos x \rightarrow du=-\sin x\,dx \rightarrow dx=\frac{du}{-\sin x}$

$=\int u^3\cdot\sin x\cdot\frac{du}{-\sin x}=-\int u^3\,du$

$=-\frac{1}{4}u^4+C$

$=-\frac{1}{4}\cos^4 x+C$

 # 14

Ans: $\frac{2}{3}\left(x+2\right)^{3/2}+C$

$\int \tan x\sec^2 x\,dx$

$u=\tan x \rightarrow du=\sec^2 x\,dx \rightarrow dx=\frac{du}{\sec^2 x}$

$=\int u\cdot\sec^2 x\cdot\frac{du}{\sec^2 x}=\int u\,du$

$=\frac{1}{2}u^2+C$

$=\frac{1}{2}\tan^2 x+C$

 # 15

Ans: $\frac{1}{2}\tan^2 x + C$

$\int \sqrt{\cos 6x} \cdot \sin 6x\, dx$

$u = \cos 6x \rightarrow du = -6\sin 6x\, dx \rightarrow dx = \frac{du}{-6\sin 6x}$

$= \int \sqrt{u} \cdot \sin 6x \cdot \frac{du}{-6\sin 6x}$

$= -\frac{1}{6}\int u^{1/2} du = -\frac{1}{6} \cdot \frac{u^{3/2}}{\frac{3}{2}} + C$

$= -\frac{1}{9}u^{3/2} + C = -\frac{1}{9}(\cos 6x)^{3/2} + C$

 # 34

Ans: $-\frac{1}{16(8x-1)^2} + C$

$\int \frac{x+4}{x^2+8x-3} dx$

$u = x^2 + 8x - 3 \rightarrow du = (2x+8)\, dx \rightarrow dx = \frac{du}{2(x+4)}$

$= \int \frac{x+4}{u} \cdot \frac{du}{2(x+4)}$

$= \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2 + 8x - 3| + C$

 # 22

Ans: $\frac{1}{12}(x^{4/3} + 9)^9 + C$

$\int e^{-x} \tan(e^{-x}) + C$

$u = e^{-x} \rightarrow du = -e^{-x}\, dx \rightarrow dx = \frac{du}{-e^{-x}}$

$= \int e^{-x} \cdot \tan u \frac{du}{-e^{-x}} = -\int \tan u\, du$

$= -(-\ln|\cos u|) + C = \ln|\cos(e^{-x})| + C$

 # 24

Ans: $3\ln\left|\sin\frac{x}{3}\right| + C$

$\int \frac{5}{4-3x} dx$

$u = 4 - 3x \rightarrow du = -3\, dx \rightarrow dx = \frac{du}{-3}$

$= \int \frac{5}{u} \cdot \frac{du}{-3}$

$= -\frac{5}{3}\int \frac{1}{u} du = -\frac{5}{3}\ln|u| + C$

$= -\frac{5}{3}\ln|4 - 3x| + C$

 # 7

Ans: $\frac{1}{24}\tan 8x + C$

$\int \frac{x^2}{5-x^3} dx$

$u = 5 - x^3 \rightarrow du = -3x^2\, dx \rightarrow dx = \frac{du}{-3x^2}$

$= \int \frac{x^2}{u} \frac{du}{-3x^2} = -\frac{1}{3}\ln|u| + C$

$= -\frac{1}{3}\ln|5 - x^3| + C$

 # 17

Ans: $\frac{3}{4}(x^2+1)^{2/3} + C$

$\int \frac{x^2 - 3x - 5}{x} dx$

$= \int \left(\frac{x^2}{x} - \frac{3x}{x} - \frac{5}{x}\right) dx = \int \left(x - 3 - \frac{5}{x}\right) dx$

$= \frac{1}{2}x^2 - 3x - 5\ln|x| + C$

12

Ans: $\ln\left|e^x - e^{-x}\right| + C$

$$\int 6x^2\sqrt{3x^3-1}\,dx$$

$$u = 3x^3 - 1 \;\rightarrow\; du = 9x^2\,dx \;\rightarrow\; dx = \frac{du}{9x^2}$$

$$= \int 6x^2 \cdot \sqrt{u} \cdot \frac{du}{9x^2} = \frac{2}{3}\int \sqrt{u}\,du$$

$$= \frac{2}{3}\frac{u^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{4}{9}\left(3x^3-1\right)^{3/2} + C$$

11

Ans: $-e^{\cos x} + C$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$$

$$u = e^x - e^{-x} \;\rightarrow\; du = \left(e^x + e^{-x}\right)dx \;\rightarrow\; dx = \frac{du}{e^x + e^{-x}}$$

$$= \int \frac{e^x + e^{-x}}{u}\frac{du}{e^x + e^{-x}} = \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln\left|e^x - e^{-x}\right| + C$$

5

Ans: $-\frac{1}{4}\ln|\cos 4x| + C$

$$\int \frac{\sin x}{(4-\cos x)^3}\,dx$$

$$u = 4 - \cos x \;\rightarrow\; du = \sin x\,dx \;\rightarrow\; dx = \frac{du}{\sin x}$$

$$= \int \frac{\sin x}{u^3}\frac{du}{\sin x} = \int u^{-3}\,du$$

$$= \frac{u^{-2}}{-2} + C = -\frac{1}{2u^2} + C = -\frac{1}{2(4-\cos x)^2} + C$$

6

Ans: $-\frac{1}{2(4-\cos x)^2} + C$

$$\int \frac{1}{3}\sec^2 8x\,dx$$

$$u = 8x \;\rightarrow\; du = 8\,dx \;\rightarrow\; dx = \frac{du}{8}$$

$$= \frac{1}{3}\int \sec^2 u\,\frac{du}{8} = \frac{1}{24}\tan u + C$$

$$= \frac{1}{24}\tan(8x) + C$$

29

Ans: $-\frac{1}{5}\cos 5x + C$

$$\int \frac{(\ln x)^5}{x}\,dx$$

$$u = \ln x \;\rightarrow\; du = \frac{1}{x}dx \;\rightarrow\; dx = x\,du$$

$$= \int \frac{u^5}{x}\cdot x\,du = \int u^5\,du$$

$$= \frac{1}{6}\cdot u^6 + C = \frac{1}{6}(\ln x)^6 + C$$

9

Ans: $-\frac{1}{2}\ln|\csc 2x + \cot 2x| + C$

$$\int \cos x \cdot 5^{\sin x}\,dx$$

$$u = \sin x \;\rightarrow\; du = \cos x\,dx \;\rightarrow\; dx = \frac{du}{\cos x}$$

$$= \int \cos x \cdot 5^u \frac{du}{\cos x} = \int 5^u\,du$$

$$= \frac{1}{\ln 5}\cdot 5^u + C = \frac{5^{\sin x}}{\ln 5} + C$$

4

Ans: $-\frac{1}{8}\cos^2(4x)+C$

or $\frac{1}{8}\sin^2(4x)+C$

$\int \tan 4x\,dx$

$u=4x \rightarrow du=4\,dx \rightarrow dx=\frac{du}{4}$

$=\int \tan u\frac{du}{4}=-\frac{1}{4}\ln|\cos u|+C$

$=-\frac{1}{4}\ln|\cos 4x|+C$

32

Ans: $\ln|4-\cos x|+C$

$\int \frac{3}{\sqrt{9-4x^2}}dx$

This integral results in an "arcsin form."

$a^2=9 \quad u^2=4x^2 \rightarrow a=3 \quad u=2x$

$du=2dx \rightarrow dx=\frac{du}{2}$

$=3\cdot\frac{1}{2}\int\frac{du}{\sqrt{a^2-u^2}}=\frac{3}{2}\arcsin\left(\frac{u}{a}\right)+C$

$=\frac{3}{2}\arcsin\left(\frac{2x}{3}\right)+C$

23

Ans: $\ln\left|\cos e^{-x}\right|+C$

$\int \cot\left(\frac{x}{3}\right)dx$

$u=\frac{x}{3} \rightarrow du=\frac{1}{3}dx \rightarrow dx=3\,du$

$=\int \cot u\cdot 3du=3\int \cot u\,du$

$=3\ln|\sin u|+C$

$=3\ln\left|\sin\frac{x}{3}\right|+C$

31

Ans: $5\ln|\ln x|+C$

$\int \frac{\sin x}{4-\cos x}dx$

$u=4-\cos x \rightarrow du=\sin x\,dx \rightarrow dx=\frac{du}{\sin x}$

$=\int\frac{\sin x}{u}\cdot\frac{du}{\sin x}=\int\frac{1}{u}du$

$=\ln|u|+C$

$=\ln|4-\cos x|+C$