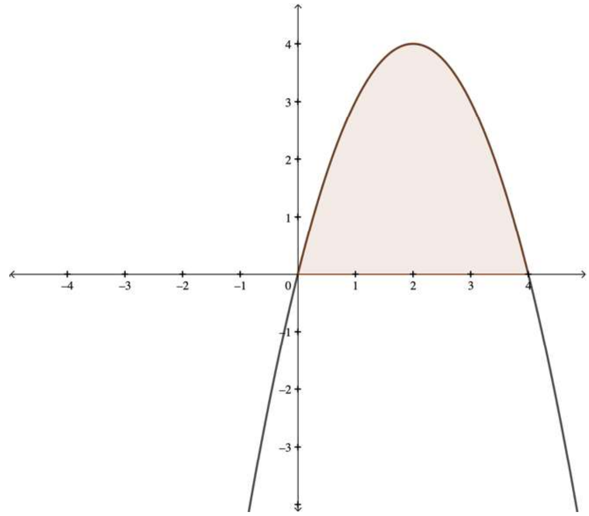
UNIT 6 (Topics 6.1-6.6) EXAM REVIEW – I Area, Riemann Sums and Definite Integrals

1.) Consider $y = 4x - x^2$.



a. Sketch the curve $y = 4x - x^2$.

b. Shade the region that represents the area of the region enclosed by $y = 4x - x^2$ and the x-axis.

c. Write the definite integral that represents the area of the region enclosed by $y = 4x - x^2$ and the x-axis.

$$\int_0^4 \left(4x - x^2\right) dx$$

d. Use the **Left** sums to **approximate** the integral found in part c on the interval [0, 4] using four subintervals.

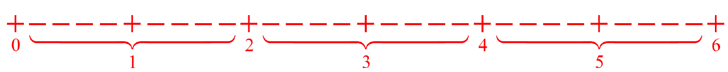
$$\Delta x = \frac{4-0}{4} = 1$$

$$\int_0^4 \left(4x - x^2\right) dx \approx A_{left} = \left(4(0)-(0)^2\right)(1) + \left(4(1)-(1)^2\right)(1) + \left(4(2)-(2)^2\right)(1) + \left(4(3)-(3)^2\right)(1)$$

$$= (0-0) + (4-1) + (8-4) + (12-9) = 3+4+1 = 10$$

e. Use the **Midpoint Rule** to **approximate** $\int_0^6 \left(4x - x^2\right) dx$ using three subintervals.

$$\Delta x = \frac{6-0}{3} = 2$$



$$\int_0^6 \left(4x - x^2\right) dx \approx A_{\text{midpoint}} = \left(4(1)-(1)^2\right)(2) + \left(4(3)-(3)^2\right)(2) + \left(4(5)-(5)^2\right)(2)$$

$$= (4-1)(2) + (12-9)(2) + (20-25)(2) = 6+6-10 = \boxed{2}$$

2.) The rate at which a wheel is turning is given by a differentiable function R (revolutions per minute) of time t (minutes). The table to the right shows the rate as measured every 3 minutes for a 12 minute period.

t	0	3	6	9	12
$R(t)$	4	5	5	10	20

Use a **Right** sum with 4 intervals to approximate $\int_0^{12} R(t)\,dt$. Using correct units, **specifically** explain the meaning of your answer in terms the motion of the wheel.

$$\int_0^{12} R(t)\,dt \approx A_{\text{right}} = R(3)(3)+R(6)(3)+R(9)(3)+R(12)(3)=(5)(3)+(5)(3)+(10)(3)+(20)(3)$$

$$=15+15+30+60=\boxed{120 \text{ revolutions}}$$

The wheel turned 120 revolutions from time $t=0$ to $t=12$ minutes.

3.) If $F(x)=\int f(x)\,dx$, $\int_2^5 f(x)\,dx=6$ and $\int_4^5 f(x)\,dx=14$, find:

a. $\int_5^5 f(x)\,dx=\boxed{0}$

b. $\int_5^4 f(x)\,dx$

$$-\int_4^5 f(x)\,dx=\boxed{-14}$$

c. $\int_2^4 f(x)\,dx=\int_2^5 f(x)\,dx-\int_4^5 f(x)\,dx$

$$=6-14=\boxed{-8}$$

d. $\int_2^5 3f(x)\,dx=3\int_2^5 f(x)\,dx$

$$=3(6)=\boxed{18}$$

e. $\int_2^5 (f(x)+2)\,dx=$

$$\int_2^5 f(x)\,dx+\int_2^5 2\,dx$$

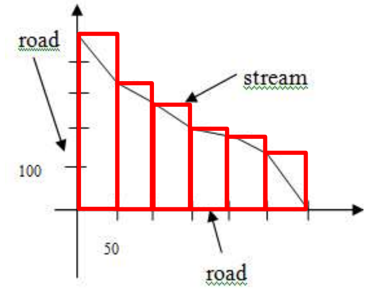
$$=6+(2)(3)=\boxed{12}$$

f. $\int_4^7 f(x-2)\,dx=\int_4^7 f(\underset{u}{x-2})\,\underset{du}{dx}$

$$=\int_{4-2}^{7-2} f(u)\,du=\int_2^5 f(u)\,du=\boxed{6}$$

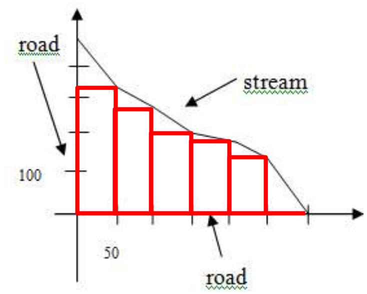
4.) The table below lists the measurements of a lot bounded by a stream and two straight roads that meet at a right angle.

x feet	0	50	100	150	200	250	300
y feet	450	362	305	268	245	156	0



a. Estimate the area of the lot using left endpoint sums where $n = 6$.

$$A_{\text{lot}} \approx A_{\text{left}} = (450)(50)+(362)(50)+(305)(50)+(268)(50)+(245)(50)+(156)(50) = \boxed{89300\ \text{ft}^2}$$

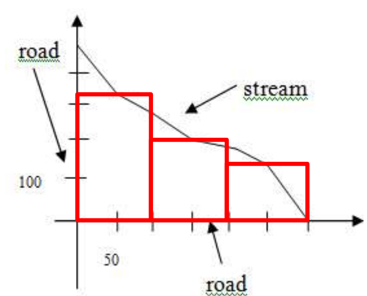


b. Estimate the area of the lot using right endpoint sums where $n = 6$.

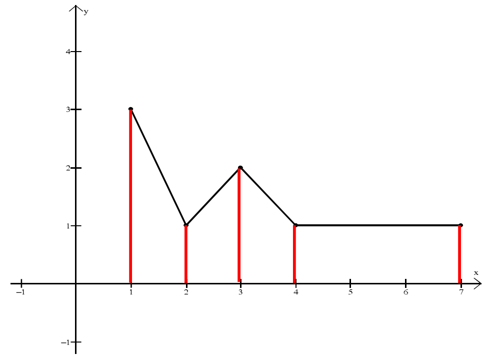
$$A_{\text{lot}} \approx A_{\text{left}} = (362)(50)+(305)(50)+(268)(50)+(245)(50)+(156)(50)+(0)(50) = \boxed{66800\ \text{ft}^2}$$

c. Estimate the area of the lot using the Midpoint Rule where $n = 3$.

$$A_{\text{lot}} \approx A_{\text{midpoint}} = (362)(100)+(268)(100)+(156)(100) = \boxed{78600\ \text{ft}^2}$$



5.) The graph of f is given in the figure to the right.



a. Evaluate $\int_1^7 f(x)\,dx$.

$$\int_1^7 f(x)\,dx = \left(\frac{3+1}{2}\right)(1) + 2\left[\left(\frac{2+1}{2}\right)(1)\right] + (3)(1) = 2+3+3 = \boxed{8}$$

b. Determine the answer if $f(x)$ is translated two units upward.

$$\int_1^7 \left(f(x)+2\right)dx = \int_1^7 f(x)\,dx + \int_1^7 2\,dx = 8 + (2)(7-1) = 8+12 = \boxed{20}$$

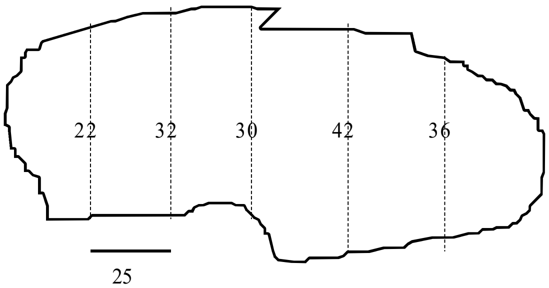
6.) Use the Trapezoid Rule to approximate the following definite integral. Round your answer to the nearest thousandths. $\int_1^5 x\sqrt{2x+1}\,dx$ for $n = 4$.

$$\int_1^5 x\sqrt{2x+1}\,dx \approx \left(\frac{\left((1)\sqrt{2(1)+1}\right)+\left((2)\sqrt{2(2)+1}\right)}{2}\right)(1) + \left(\frac{\left((2)\sqrt{2(2)+1}\right)+\left((3)\sqrt{2(3)+1}\right)}{2}\right)(1)$$

$$+\left(\frac{\left((3)\sqrt{2(3)+1}\right)+\left((4)\sqrt{2(4)+1}\right)}{2}\right)(1) + \left(\frac{\left((4)\sqrt{2(4)+1}\right)+\left((5)\sqrt{2(5)+1}\right)}{2}\right)(1)$$

$$= 33.5669\ldots \approx \boxed{33.567}$$

7.) For the following measurements (each are in feet) made across the lake shown. Use the Trapezoid Rule to approximate the area of the lake. Round your answer to the nearest square foot.



$$A_{\text{lake}} \approx A_{\text{trapezoid}} = \left(\frac{0+22}{2}\right)(25) + \left(\frac{22+32}{2}\right)(25)$$

$$+\left(\frac{32+30}{2}\right)(25) + \left(\frac{30+42}{2}\right)(25) + \left(\frac{42+36}{2}\right)(25) + \left(\frac{36+0}{2}\right)(25)$$

$$= \frac{25}{2}\left[0+2(22)+2(32)+2(30)+2(42)+2(36)\right]$$

$$= 25\left[(22)+(32)+(30)+(42)+(36)\right] = \boxed{4050\,\text{ft}^2}$$

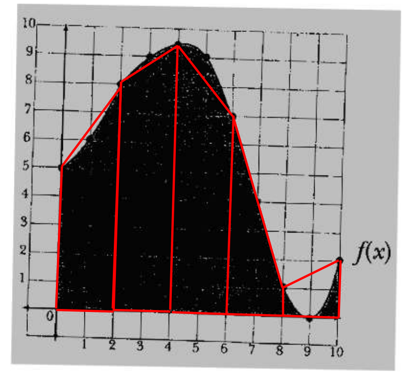
8.) Consider the graph of a continuous function $f(x)$ given to the right.

a. Approximate $\int_0^{10} f(x)\,dx$ using the Trapezoid rule with n = 5.

$$\int_0^{10} f(x)\,dx \approx A_{\text{Trap}} = \left(\frac{5+8}{2}\right)(2)+\left(\frac{8+9}{2}\right)(2)+\left(\frac{9+7}{2}\right)(2)$$

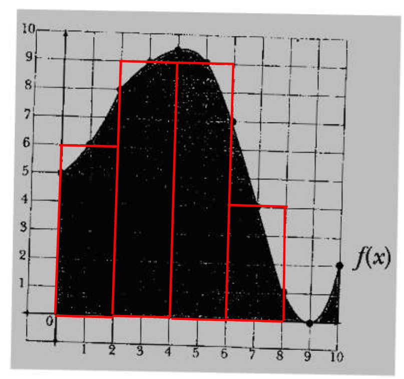
$$+\left(\frac{7+1}{2}\right)(2)+\left(\frac{1+2}{2}\right)(2)$$

$$=5+2(8)+2(9)+2(7)+2(1)+2=\boxed{57}$$



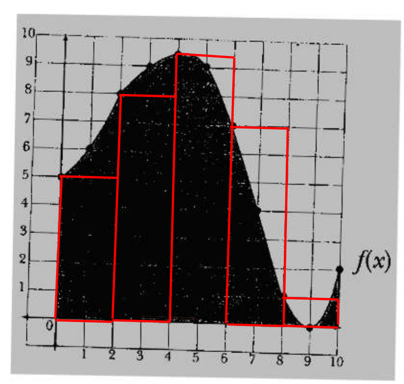
b. Approximate $\int_0^{8} f(x)\,dx$ using the Midpoint rule with n = 4.

$$\int_0^{8} f(x)\,dx \approx A_{\text{midpoint}} = (6)(2)+(9)(2)+(9)(2)+(4)(2)=\boxed{56}$$



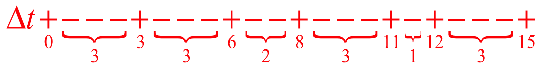
c. Approximate $\int_0^{10} f(x)\,dx$ using the left side rectangles with n = 5.

$$\int_0^{10} f(x)\,dx \approx A_{\text{left}} = (5)(2)+(8)(2)+\underbrace{(9.5)}_{\text{estimated from graph}}(2)+(7)(2)+(1)(2)=\boxed{61}$$



9.) Due to a bad storm on a low-lying road, a large circular puddle of water forms. The area of the puddle increases as the storm intensifies. The radius of the puddle, in feet, is modeled by the twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 15$, the graph of r is concave up. The table below gives selected values of the rate of change, $r'(t)$ of the radius of the puddle over the time interval $0 \le t \le 15$. The radius of the puddle is 7 feet when $t = 6$.

t (minutes)	0	3	6	8	11	12	15
$r'(t)$ (feet per minute)	1.2	2.3	3.4	4.3	4.9	5.0	6.2



a.) Use a right Riemann sum with six intervals using the data in the table to approximate $\int_0^{15} r'(t)\,dt$.

$$\int_0^{15} r'(t)\,dt \approx A_{\text{right}} = (2.3)(3)+(3.4)(3)+(4.3)(2)+(4.9)(3)+(5.0)(1)+(6.2)(3) = \boxed{64 \text{ ft}}$$

b.) Use a left Riemann sum with six intervals using the data in the table to approximate $\int_0^{15} r'(t)\,dt$.

$$\int_0^{15} r'(t)\,dt \approx A_{\text{left}} = (1.2)(3)+(2.3)(3)+(3.4)(2)+(4.3)(3)+(4.9)(1)+(5.0)(3) = \boxed{50.1 \text{ ft}}$$

c.) Use a Trapezoidal sum with six intervals using the data in the table to approximate $\int_0^{15} r'(t)\,dt$.

$$\int_0^{15} r'(t)\,dt \approx A_{\text{trap}} = \left(\frac{1.2+2.3}{2}\right)(3)+\left(\frac{3.4+2.3}{2}\right)(3)+\left(\frac{3.4+4.3}{2}\right)(2)+\left(\frac{4.9+4.3}{2}\right)(3)$$
$$+\left(\frac{4.9+5.0}{2}\right)(1)+\left(\frac{6.2+5.0}{2}\right)(3) = \boxed{57.05 \text{ ft}}$$

d.) Use a Midpoint sum with three intervals using the data in the table to approximate $\int_0^{15} r'(t)\,dt$.

$$\int_0^{15} r'(t)\,dt \approx A_{\text{mid}} = r'(3)(6-0)+r'(8)(11-6)+r'(12)(15-11) = (2.3)(6)+(4.3)(5)+(5.0)(4) = \boxed{55.3 \text{ ft}}$$

e.) Using correct units, explain the meaning of $\int_0^{15} r'(t)\,dt$ in the context of the problem situation.

$\int_0^{15} r'(t)\,dt$ gives the total amount the radius of the puddle changes from time $t = 0$ to $t = 15$ minute