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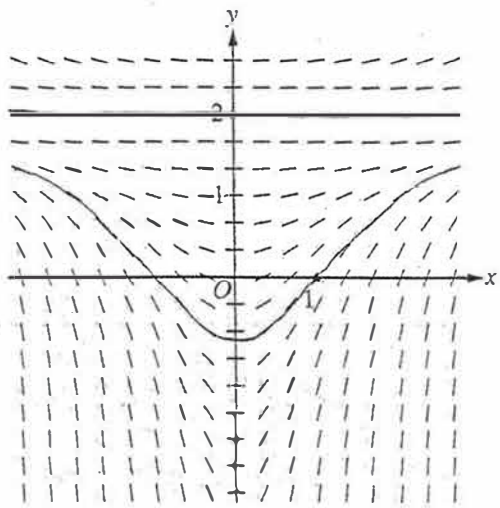
6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$y = m_T(x-1) + 0$

$m_T = \frac{1}{3} \cdot 1(0-2)^2$

$= \frac{1}{3}(4)$

$= 4/3$

$y = 4/3\,(x-1)$

$f(0.7) \approx 4/3\,(0.7-1)$

$\approx 4/3\left(-\frac{3}{10}\right)$

$\approx -4/10 = -2/5$

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(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$u = y - 2$

$du = dy$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3} x \, dx$$

$$\int \frac{du}{u^2} = \frac{1}{3} \int x \, dx$$

$$-\frac{1}{u} = \frac{1}{3}\left(\frac{x^2}{2}\right) + C$$

$$-\frac{1}{(y-2)} = \frac{x^2}{6} + C$$

point: $x = 1$
$y = 0$

$$C = \frac{1}{2} - 1/6 = 3/6 - 1/6 = 2/6 = 1/3$$

$$\therefore \frac{-1}{y-2} = \frac{x^2}{6} + 1/3$$

$$\frac{1}{y-2} = -\frac{x^2}{6} - 1/3$$

$$y - 2 = \left(\frac{1}{-\frac{x^2}{6} - \frac{2}{6}}\right) \frac{6}{6}$$

$$y - 2 = \frac{6}{-(x^2+2)}$$

$$y = \frac{6}{-(x^2+2)} + 2$$

GO ON TO THE NEXT PAGE.