Calculus

Write your questions and thoughts here!

1.2 Limits Analytically

Name:____________________________

Notes

Recall: What is a limit?

The y-value a function approaches at a given x-value.

Finding a limit:

1. Direct Substitution.
2. Simplify and then try substitution.
   a. Factor and cancel.
   b. Rationalize if you see square roots.
3. L'Hosital's Rule (for indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$) UNIT 4

most common

Direct Substitution		Factor and Cancel	
1. $\lim_{x\to -1}(x^2+2x-4)$ $=(-1)^2+2(-1)-4$ $=1-2-4$ $=1-6$ $=-5$	2. $\lim_{x\to 2}\sqrt{3x-2}$ $=\sqrt{3(2)-2}$ $=\sqrt{6-2}$ $=\sqrt{4}$ $=2$	3. $\lim_{x\to 0}\frac{4x^2-5x}{x}$ $=\lim_{x\to 0}\frac{x(4x-5)}{x}$ $=\lim_{x\to 0}(4x-5)$ $=4(0)-5$ $=-5$	4. $\lim_{x\to -7}\frac{2x^2+13x-7}{x+7}$ $=\lim_{x\to -7}\frac{(x+7)(2x-1)}{x+7}$ $=\lim_{x\to -7}(2x-1)$ $=2(-7)-1$ $=-14-1$ $=-15$

Rationalize	Two variables
5. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$ $\frac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)}$	6. $\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$

$$=\lim_{x\to 5}\frac{(\sqrt{x+4})^2-(3)^2}{(x-5)(\sqrt{x+4}+3)}$$
$$=\lim_{x\to 5}\frac{x+4-9}{(x-5)(\sqrt{x+4}+3)}$$
$$=\lim_{x\to 5}\frac{x-5}{(x-5)(\sqrt{x+4}+3)}$$
$$=\lim_{x\to 5}\frac{1}{\sqrt{x+4}+3}$$
$$=\frac{1}{\sqrt{5+4}+3}$$
$$=\frac{1}{\sqrt{9}+3}$$
$$=\frac{1}{3+3}$$
$$=\frac{1}{6}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2hx+h^2-\cancel{3x}-3h-\cancel{x^2}+\cancel{3x}}{h}$$
$$=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$$
$$=\lim_{h\to 0}(2x+h-3)$$
$$=2x+(0)-3$$
$$=2x-3$$

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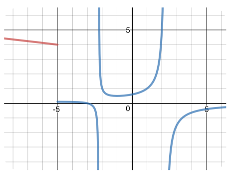
1.2 Limits Analytically

Notes



Piecewise defined functions and limits

$$f(x)=\begin{cases}\sqrt{11-x}, & x<-5\\ \dfrac{x+3}{5-x^2}, & x\ge -5\end{cases}$$

$f(-5)=\sqrt{11-(-5)}$
$=\sqrt{16}$
$=4$

$f(5)=\frac{-5+3}{5-(-5)^2}=\frac{-2}{5-25}=\frac{-2}{-20}$

7. $\lim_{x\to -5^-} f(x) = 4$

8. $\lim_{x\to -5^+} f(x) = \frac{1}{10}$

9. $\lim_{x\to -5} f(x) =$ d.n.e.

$$g(x)=\begin{cases}\sqrt{10-x^2}, & x<-1\\ \dfrac{26-5x^2}{7}, & -1<x\le e\\ \ln x^3, & x>e\end{cases}$$

$g(e)=\ln e^3$
$=3\ln e$
$=3\cdot 1$
$=3$

10. $\lim_{x\to -1} g(x) = 3$

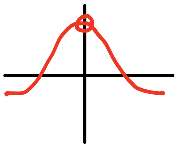
$\lim_{x\to -1^-} g(x)=3$; $\lim_{x\to -1^+} g(x)=3$

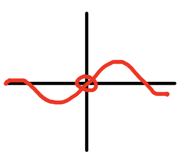
11. $\lim_{x\to e^+} g(x) = 3$

12. $\lim_{x\to e} g(x) =$ DNE

$g(e)=\frac{26-5e^2}{7}$

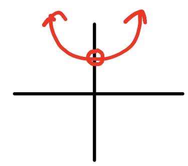
DONT NEED TO KNOW



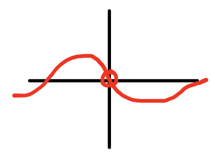


Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x} = 1$$



$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0 \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$



13. $\lim_{x\to 0}\frac{\sin 3x}{x}\cdot\frac{3}{3}$

$=3\lim_{x\to 0}\frac{\sin 3x}{3x}$

$=3\cdot 1$

$=3$

14. $\lim_{x\to 0}\frac{\tan 4x}{8x}=\lim_{x\to 0}\frac{\sin 4x}{\cos 4x}\cdot\frac{1}{8x}$

$=\lim_{x\to 0}\frac{\frac{1}{2}\sin 4x}{\frac{1}{2}8x}\cdot\frac{1}{\cos 4x}$

$=\frac{1}{2}\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{1}{\cos 4x}$

$=\frac{1}{2}\cdot 1\cdot\frac{1}{\cos 4(0)}$

$=\frac{1}{2}\cdot 1\cdot\frac{1}{\cos 0}$

$=\frac{1}{2}\cdot 1\cdot\frac{1}{1}$

$=\frac{1}{2}$

15. $\lim_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)}$

$=\lim_{x\to 0}\frac{(\cos x-1)(\cos x+1)}{x(\cos x+1)}$

$=\lim_{x\to 0}\frac{\cos x-1}{x}$

$=0$

16. $\lim_{x\to 0}\frac{\sin 7x}{\sin 9x}$

$=\frac{7}{9}\lim_{x\to 0}\frac{\sin 7x}{7\cdot x}\cdot\frac{1\cdot 9x}{\sin 9x}$

$=\frac{7}{9}\lim_{x\to 0}\frac{\sin 7x}{7x}\cdot\frac{9x}{\sin 9x}$

$=\frac{7}{9}\cdot 1\cdot 1$

$=\frac{7}{9}$

Now summarize what you learned!