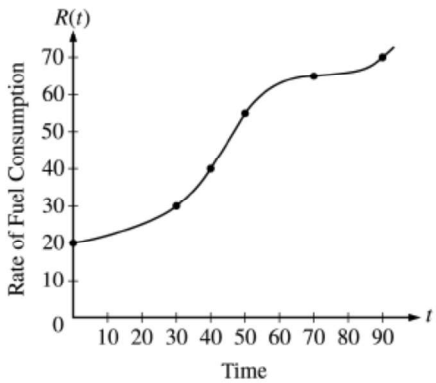
2003 AP® CALCULUS AB **Problem #3**



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

$$R'(45) = \frac{R(40) - R(50)}{40 - 50}$$

$$= \frac{40 - 55}{-10} \quad +1$$

$$= \frac{-15}{-10}$$

$$= 3/2 \text{ gallons/min}^2 \quad +1$$

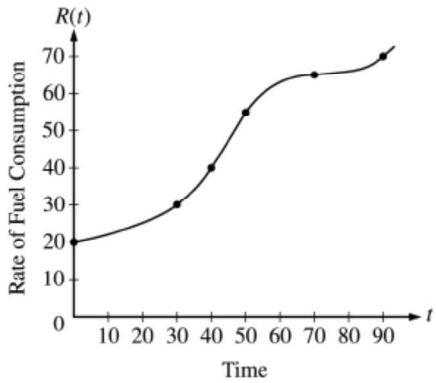
(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

$R(t)$ is increasing fastest at $t = 45$.

$\therefore R'(t)$ is a relative max at $t = 45$ +1

$\therefore R''(t) = 0$ when $t = 45$ min. +1

2003 AP® CALCULUS AB Problem #3 continued



t (minutes)	$R(t)$ (gallons per minute)
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(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

$\int_0^{90} R(t)dt \approx 30(20) + 10(30) + 10(40) + 20(55) + 20(65)$

$\approx 600 + 300 + 400 + 1100 + 1300$

≈ 3700 gallons +1

3700 gallons is less than the value of $\int_0^{90} R(t)dt$ +1

b/c the graph of $R(x)$ is increasing on the interval.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

+1/2

$\int_0^{b} R(t)dt$ is the total fuel consumption (in gallons) for the first "b" minutes.

+1

$\frac{1}{b}\int_0^{b} R(t)dt$ is the average value of fuel consumption (in gallons/min) +1/2

during the first b minutes +1